Games and Economic Behavior 108 (2018) 182–205

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Polluted river problems and games with a permission structure ☆

René van den Brink [a], Simin He [b,1], Jia-Ping Huang [c,*,2]

[a] Department of Econometrics and Tinbergen Institute, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands
[b] School of Economics and Key Laboratory of Mathematical Economics, Shanghai University of Finance and Economics, 111 Wuchuan Rd, Shanghai 200433, PR China
[c] China Center for Special Economic Zone Research, Shenzhen University, Nanhai Ave 3688, Shenzhen, Guangdong 518060, PR China

ARTICLE INFO

Article history:
Received 30 August 2016
Available online 18 October 2017

JEL classification:
C71
D61
D62

Keywords:
Polluted river
Cost sharing
Axiomatization
Permission values

ABSTRACT

When a polluted river passes through several different regions, a challenging question is how should the costs for cleaning the river be shared among the regions. Following Ni and Wang (2007) and Dong et al. (2012), we first show that the *Upstream Equal Sharing method* and the *Downstream Equal Sharing method* coincide with the *conjunctive permission value* (van den Brink and Gilles, 1996) of an associated game with *a permission structure*, which is obtained as the *Shapley value* of an associated restricted game. Two main advantages of this approach are (i) we obtain new axiomatizations of the two sharing methods based on axiomatizations of the conjunctive permission value, and (ii) by applying the alternative *disjunctive permission value*, obtained as the Shapley value of a different restricted game, we propose the new Upstream Limited Sharing method and provide an axiomatization.

1. Introduction

The allocation of (clean) river water has gained attention in the recent literature. In particular, there is a growing literature on applying game theory to such allocation problems, see e.g. Ambec and Sprumont (2002), Parrachino et al. (2006), van den Brink et al. (2007), Ambec and Ehlers (2008), Khmelnitskaya (2010), Wang (2011), Ansink and Weikard (2012), van den Brink et al. (2012) and van den Brink et al. (2014b). Typically, the goal is to obtain an efficient allocation of water over the agents along the river, where water can stream from upstream to downstream agents against a possible monetary compensation from downstream to upstream agents to support this allocation.

Besides the allocation of available river water, Ni and Wang (2007) introduced a model of a situation where a river is polluted, and in order to consume the water cleaning costs must be made to clean the water. When the river passes through several different countries or regions, a natural question is how should the costs be shared among the agents. An extreme solution is that each country just pays for the cleaning cost at its own region. However, if upstream countries are also partly responsible for the pollution at a certain river segment, then it seems reasonable that upstream countries share

☆ We thank Gerard van der Laan and two anonymous referees for comments on a previous version.
* Corresponding author.
E-mail addresses: jrbrink@feweb.vu.nl (R. van den Brink), he.simin@mail.shufe.edu.cn (S. He), huangjp@szu.edu.cn (J.-P. Huang).
[1] This author thanks CREED, the University of Amsterdam in the Netherlands. This paper was mostly written while she did her PhD there.
[2] This author is supported by NWO grant 400-09-434, grant 16QNFC45 from Shenzhen University, and the Young Creative Talent Project (grant 2016WQNCX129) from the Department of Education of Guangdong Province, China.

https://doi.org/10.1016/j.geb.2017.10.005
0899-8256/

in the pollution cost of their downstream countries. On the other hand, since downstream countries benefit from upstream countries cleaning the river, it might be reasonable that downstream countries contribute in the cleaning cost of upstream countries.

These issues are considered initially by Ni and Wang (2007) for single spring rivers, and generalized by Dong et al. (2012) for rivers with multiple springs. They introduced the so-called *cost sharing problem on a river network*, shortly called *polluted river problem*, where besides a river structure, for every river segment a fixed cleaning cost is given.[3] They introduce and axiomatize three cost sharing methods reflecting the three different forms of responsibility mentioned above: the Local Responsibility Sharing (LRS) method, the Upstream Equal Sharing (UES) method and the Downstream Equal Sharing (DES) method. They also show that these methods can be obtained as the *Shapley value* of associated games.

In this paper, we first show that the UES and DES methods coincide with the *conjunctive permission value* (Gilles et al., 1992; van den Brink and Gilles, 1996) of an associated *game with a permission structure*. Games with a permission structure model situations where players in a cooperative transferable utility game belong to some hierarchical structure where players need permission from some of their superiors before they can cooperate with other players.[4] The polluted river problems correspond to games with a permission structure where the game is the inessential game where the worth of each coalition is the sum of the cleaning costs for all agents in the coalition (which is the Local Responsibility game used by Dong et al. (2012) to obtain the LRS method), and the digraph (permission structure) is the sink tree corresponding to the river structure with the arcs oriented from upstream to downstream agents.

One of the best known solutions for games with a permission structure is the conjunctive permission value which is obtained as the Shapley value of an associated restricted game. After establishing that the UES method can be obtained as conjunctive permission value, we apply the axiomatization of the conjunctive permission value of van den Brink and Gilles (1996) to the class of polluted river problems. We show that this yields an axiomatization of the UES method and discuss the differences and similarities with that of Dong et al. (2012). Comparing these two axiomatic systems, we find that the advantage of introducing an axiomatization by games with a permission structure is threefold: (i) it splits one axiom into two other axioms that each express a different allocation principle, (ii) by putting it in a more general context, we will see that new axiomatizations and even new cost sharing methods appear, and (iii) we can do without a strong independence axiom. Also, it turns out that the axioms have a good interpretation in terms of water allocation principles in International Water Law.

Kilgour and Dinar (1995) studied general principles to resolve water allocation disputes resulting from International Water Law, which leads a direction of the implications of the method. Two important principles are Absolute Territorial Sovereignty (also known as the Harmon doctrine) and Territorial Integration of all Basin States. Absolute Territorial Sovereignty (ATS) states that every country has the absolute sovereignty over the inflow of the river on its own territory. Territorial Integration of all Basin States (TIBS) states that 'the water of an international watercourse belongs to all basin states combined, no matter where it enters the watercourse. It does not make any country the legal owner of water. Each basin state is entitled to a reasonable and equitable share in the optimal use of the available water' (see Lipper, 1967 and McCaffrey, 2001). A problem with water allocation principles as described above is that often they can be interpreted in several ways, or are in conflict with each other. For example, there seems to be a tension between the ATS and TIBS principles, where ATS allows upstream countries to fully claim the water on their own territory but TIBS allows downstream countries to have a claim on the optimal water use of the upstream water. Notice that TIBS refers to the 'optimal use' of river water, so even when the upstream countries claim 'their' water, question still is how to allocate the increase in welfare when water is sent downstream. Whereas TIBS just mentions that each basin state is entitled to a reasonable and equitable share in the optimal use of the available water, it does not say anything about what are these equitable and reasonable shares. This is exactly where cooperative game theory comes in since a main goal of solutions for cooperative games, such as the Shapley value, is to determine such equitable and reasonable shares.

Another advantage of studying the UES method as a conjunctive permission value for a specific class of games with a permission structure is that other axiomatizations of the conjunctive permission value can be applied. In this way, we find a new axiomatization of the UES method by applying the axiomatization in van den Brink (1999) yielding a new axiom for polluted river problems. This new axiom is called *externality fairness* and reflects what happens if one agent stops to participate in the cleaning cost agreement among the agents. Specifically, consider an agent i and its downstream neighbor j. Suppose that the sub-river consisting of agent i and all its upstream agents retreat from the agreement. Then the cooperation structure splits in two components: i and its upstream agents, and j with all the other agents. Each of the two components now only pay their own cost and do not contribute anymore in the cleaning cost of the other component. In particular, agent i does not pay anymore for its downstream neighbor j and the other agents in j's component. Externality fairness requires that in this case the change (increase) of the contribution of j in the cost of its component (in the new cooperation structure) should be equal to the change in the contribution of any of its other upstream neighbors. So, the refusal of an upstream neighbor i of j to contribute to the cleaning cost in the river component with j, affects the contributions of the other upstream neighbors of j by the same amount as j.

[3] Alcalde-Unzu et al. (2015) extended this model by having transfer rates about how pollution flows through the river, so one can take more precise care about who is responsible for the pollution in a river segment.

[4] Another type of authority, based on command, is considered in Hu and Shapley (2003).

Another advantage of the relation between polluted river problems and games with a permission structure is that other solutions for games with a permission structure can be applied. For example, by applying the *disjunctive permission value* of Gilles and Owen (1994), obtained as the Shapley value of a different restricted game, we obtain a new cost sharing method, called the *Upstream Limited Sharing* (ULS) method. According to this method, the cleaning cost on a river segment is allocated to the upstream agents, but their contribution is not necessarily equal as it is in the UES method. We apply the axiomatization of the disjunctive permission value of van den Brink (1997) to obtain an axiomatization of this new cost sharing method, yielding a new axiom, which is called *participation fairness*, and reflects what happens if one agent stops to participate in the cleaning cost agreement among all agents in a different way than externality fairness of the UES method. To be more specific, again consider an agent i and its downstream neighbor j, and suppose again that the sub-river consisting of i and all its upstream agents retreat from the agreement. As mentioned above, the cooperation structure splits in two components: i and its upstream agents, and j with all the other agents. *Participation fairness* requires the change in the contribution of i and j after breaking the agreement to be equal. In other words, the refusal of an upstream neighbor of j to contribute to the cleaning cost in the river component with j, affects j and this upstream neighbor by the same amount. We also show that the ULS method can be obtained as the Shapley value of another newly defined game on the polluted river problems with multiple springs. This result provides an alternative (direct) definition of the ULS method.

Finally, by reversing the orientation of the arcs in the permission structure, orienting them from downstream to upstream, and applying the conjunctive permission value we obtain the DES method. Since for games with a permission structure where the permission structure is a rooted tree, the conjunctive and disjunctive permission values coincide, the DES method can also be obtained as the disjunctive permission value of the associated game with a permission structure. Also for this method, we obtain an axiomatization from the literature on games with a permission structure and compare it with the one of Dong et al. (2012).

This paper illustrates the strength of the Shapley value, not only as a solution for cooperative games, but also its appealing properties for applications. As mentioned, Ni and Wang (2007) and Dong et al. (2012) obtained the UES and DES methods as the Shapley value of associated games. The axioms underlying these methods are essential features of the Shapley value. In this paper we show that the UES and DES methods also can be obtained as the Shapley value of alternative games yielding an alternative axiomatization. Again, these axioms are related to essential features of the Shapley value. For example, fairness is an implicit feature of the Shapley value (see van den Brink, 2001 and Myerson, 1977). Moreover, another strength of the Shapley value is that by considering other games associated to the polluted river problem, we can obtain alternative cost sharing methods, such as the ULS method, which by being a Shapley value, gives a natural comparison with other methods.

The paper is organized as follows. Section 2 contains preliminaries on games with a permission structure (being the tool that we will use) and polluted river problems (being the allocation problem to which we will apply this tool). In Section 3, we show that the UES method coincides with the conjunctive permission value of an associated game with a permission structure, and provide axiomatizations. In Section 4, we apply the disjunctive permission value yielding the new ULS method for polluted river problems, and provide an axiomatization. In Section 5, we show that by reversing the orientation of the arcs we obtain the DES method as conjunctive as well as disjunctive permission value. We end with concluding remarks.

2. Preliminaries

2.1. Cooperative TU-games and digraphs

2.1.1. TU-games

A situation in which a finite set of players $N \subset \mathbb{N}$ can generate certain payoffs by cooperation can be described by a *cooperative game with transferable utility* (or simply a TU-game), being a pair (N, v) where $v : 2^N \to \mathbb{R}$ is a *characteristic function* on N satisfying $v(\emptyset) = 0$. For every coalition $S \subseteq N$, $v(S) \in \mathbb{R}$ is the *worth* of coalition S, i.e. the members of coalition S can obtain a total payoff of $v(S)$ by agreeing to cooperate. If there is no confusion about the player set, we denote a TU-game (N, v) just by its characteristic function v. We denote the collection of all TU-games by $\mathcal{G}$.

A *payoff vector* for game $(N, v) \in \mathcal{G}$ is an $|N|$-dimensional vector $x \in \mathbb{R}^N$ assigning a payoff $x_i \in \mathbb{R}$ to any player $i \in N$. A (single-valued) *solution* for TU-games is a function f that assigns a payoff vector $f(N, v) \in \mathbb{R}^N$ to every TU-game $(N, v) \in \mathcal{G}$. One of the most famous solutions for TU-games is the *Shapley value* (Shapley, 1953) given by

$$Sh_i(N, v) = \sum_{S \subseteq N : i \in S} \frac{(|S| - 1)!(|N| - |S|)!}{|N|!} \left(v(S) - v(S \setminus \{i\})\right).$$

A game v is *additive* or *inessential* if $v(S) = \sum_{i \in S} v(\{i\})$ for all $S \subseteq N$.

2.1.2. Digraphs

A directed graph or *digraph* is a pair (N, D) where $N \subset \mathbb{N}$ is a finite set of nodes (representing the players) and $D \subseteq N \times N$ is a binary relation on N. We assume the digraph to be irreflexive, i.e., $(i, i) \notin D$ for all $i \in N$. Again, if there is no confusion about the set of nodes N, we denote a digraph (N, D) just by its binary relation D. We denote the collection of all irreflexive digraphs by $\mathcal{D}$. For $i \in N$, the nodes in $P_D(i) := \{j \in N : (j,i) \in D\}$ are called the *predecessors* of i in D, and the nodes in $P_D^{-1}(i) := \{j \in N : i \in P_D(j)\} = \{j \in N : (i,j) \in D\}$ are called the *successors* of i. For given $(N, D) \in \mathcal{D}$, a (directed) *path* from i to j is a sequence of distinct nodes $(h_1, \ldots, h_t)$ such that $h_1 = i$, $h_{k+1} \in P_D^{-1}(h_k)$ for $k = 1, \ldots, t-1$, and $h_t = j$. We call $j \in N$ a *superior* of $i \in N$ in digraph (N, D) if there is a directed path from j to i. We denote the set of superiors of i by $\widehat{P}_D(i)$. We call the players in the set $\widehat{P}_D^{-1}(i) = \{j \in N : i \in \widehat{P}_D(j)\} = \{j \in N$: there is a directed path from i to $j\}$ the *subordinates* of i in digraph (N, D). For a set of players $S \subseteq N$, we denote by $P_D(S) = \bigcup_{i \in S} P_D(i)$, respectively, $P_D^{-1}(S) = \bigcup_{i \in S} P_D^{-1}(i)$, the sets of predecessors, respectively successors of players in coalition S. Also, for $S \subseteq N$, we denote $\widehat{P}_D(S) = \bigcup_{i \in S} \widehat{P}_D(i)$ and $\widehat{P}_D^{-1}(S) = \bigcup_{i \in S} \widehat{P}_D^{-1}(i)$.

A directed path $(i_1, \ldots, i_t)$, $t \geq 2$, in D is a *cycle* in D if $(i_t, i_1) \in D$. We call digraph D *acyclic* if it does not contain any cycle. We denote the class of all acyclic digraphs by $\mathcal{D}_A$. Note that acyclicity of digraph D implies that D has at least one node that does not have a predecessor, and at least one node that does not have a successor. We denote $T(D) = \{i \in N : P_D(i) = \emptyset\}$ the set of (top) nodes that do not have a predecessor, and $B(D) = \{i \in N : P_D^{-1}(i) = \emptyset\}$ the set of (bottom) nodes that do not have a successor.

A digraph $(N, D) \in \mathcal{D}$ is a *rooted tree* if and only if there is an $i_0 \in N$ such that (i) $T(D) = \{i_0\}$, (ii) $\widehat{P}_D^{-1}(i_0) = N \setminus \{i_0\}$, and (iii) $|P_D(i)| = 1$ for all $i \in N \setminus \{i_0\}$. In this case, i_0 is called the *root* of the tree. Note that this implies that D is acyclic.

A digraph $(N, D) \in \mathcal{D}$ is a *sink tree* if and only if there is an $i_s \in N$ such that (i) $B(D) = \{i_s\}$, (ii) $\widehat{P}_D(i_s) = N \setminus \{i_s\}$, and (iii) $|P_D^{-1}(i)| = 1$ for all $i \in N \setminus \{i_s\}$. Note that this also implies that D is acyclic. In this case, i_s is called the *sink* of the tree.

2.2. Games with a permission structure

A game with a permission structure describes a situation where some players in a TU-game need permission from other players before they are allowed to cooperate within a coalition. A permission structure can be described by a directed graph on N.

A triple (N, v, D) with $N \subset \mathbb{N}$ a finite set of players, $(N, v) \in \mathcal{G}$ a TU-game and $(N, D) \in \mathcal{D}$ a digraph on N is called a *game with a permission structure*. We denote by $\mathcal{GP}$ the collection of all games with a permission structure.

In the *conjunctive approach* as introduced in Gilles et al. (1992) and van den Brink and Gilles (1996) it is assumed that a player needs permission from all its predecessors in order to cooperate with other players. Therefore, a coalition is feasible if and only if for every player in the coalition all its predecessors are also in the coalition. So, for permission structure D the set of *conjunctive feasible coalitions* is given by

$$\Phi_D^c = \{S \subseteq N : P_D(i) \subseteq S \text{ for all } i \in S\}.$$

Since Φ_D^c is union closed, i.e. the union of any two feasible coalitions is also feasible, every coalition has a unique largest feasible subset. The induced *conjunctive restricted* game of the game with permission structure (N, v, D) assigns to each coalition $S \subseteq N$ the worth of its largest conjunctive feasible subset, i.e. it is the game $r_{v,D}^c : 2^N \to \mathbb{R}$, given by

$$r_{v,D}^c(S) = v\left(\bigcup_{\{T \in \Phi_D^c : T \subseteq S\}} T\right) = v\big(\{i \in S : \widehat{P}_D(i) \subseteq S\}\big) \quad \text{for all } S \subseteq N. \tag{2.1}$$

Then the *conjunctive permission value* φ^c is the solution that assigns to every game with a permission structure the Shapley value of the conjunctive restricted game, thus

$$\varphi^c(N, v, D) = Sh(N, r_{v,D}^c) \text{ for all } (N, v, D) \in \mathcal{GP}.$$

Alternatively, in the *disjunctive approach* to acyclic permission structures, as introduced in Gilles and Owen (1994) and van den Brink (1997), it is assumed that a player needs permission from at least one of its predecessors (if it has any) in order to cooperate with other players. Therefore a coalition is feasible if and only if for every player in the coalition at least one of its predecessors (if it has any) is also in the coalition. So, for permission structure D the set of *disjunctive feasible coalitions* is given by

$$\Phi_D^d = \{S \subseteq N : P_D(i) \cap S \neq \emptyset \text{ for all } i \in S \setminus T(D)\}.$$

Again, by union closedness of Φ_D^d, we can define the induced *disjunctive restricted* game of the game with permission structure (N, v, D) as the game that assigns to each coalition $S \subseteq N$ the worth of its largest disjunctive feasible subset, i.e. it is the game $r_{v,D}^d : 2^N \to \mathbb{R}$, given by

$$r_{v,D}^d(S) = v\left(\bigcup_{\{T \in \Phi_D^d : T \subseteq S\}} T\right) \text{ for all } S \subseteq N. \tag{2.2}$$

Then the *disjunctive permission value* φ^d is the solution that assigns to every game with a permission structure the Shapley value of the disjunctive restricted game, thus

$$\varphi^d(N,v,D) = Sh(N, r^d_{v,D}) \text{ for all } (N,v,D) \in \mathcal{GP}. \tag{2.3}$$

Player $i \in N$ is *inessential* in game with permission structure (N,v,D) if i and all its subordinates are *null* players in (N,v), i.e., if $v(S) = v(S \setminus \{j\})$ for all $S \subseteq N$ and $j \in \{i\} \cup \widehat{P}_D^{-1}(i)$. Player $i \in N$ is called *necessary* in game (N,v) if $v(S) = 0$ for all $S \subseteq N \setminus \{i\}$. A TU-game $(N,v) \in \mathcal{G}$ is *monotone* if $v(S) \le v(T)$ for all $S \subseteq T \subseteq N$. The class of all monotone games is denoted by $\mathcal{G}_M$. Next we recall some axiomatizations of the permission values.[5]

Efficiency For every $(N,v,D) \in \mathcal{GP}$, it holds that $\sum_{i \in N} f_i(N,v,D) = v(N)$.

Additivity For every $(N,v,D), (N,w,D) \in \mathcal{GP}$, it holds that $f(N,v+w,D) = f(N,v,D) + f(N,w,D)$, where $(v+w)$ is given by $(v+w)(S) = v(S) + w(S)$ for all $S \subseteq N$.

Inessential player property For every $(N,v,D) \in \mathcal{GP}$, if $i \in N$ is an inessential player in (N,v,D) then $f_i(N,v,D) = 0$.

Necessary player property For every $(N,v,D) \in \mathcal{GP}$ with $(N,v) \in \mathcal{G}_M$, if $i \in N$ is a necessary player in (N,v) then $f_i(N,v,D) \ge f_j(N,v,D)$ for all $j \in N$.

Structural monotonicity For every $(N,v,D) \in \mathcal{GP}$ with $(N,v) \in \mathcal{G}_M$, if $i \in N$ and $j \in P_D^{-1}(i)$ then $f_i(N,v,D) \ge f_j(N,v,D)$.

These five axioms characterize the conjunctive permission value.

Theorem 2.1 *(van den Brink and Gilles, 1996). A solution f on $\mathcal{GP}$ is equal to the conjunctive permission value φ^c if and only if it satisfies efficiency, additivity, the inessential player property, the necessary player property and structural monotonicity.*

On the class of games with an acyclic permission structure, from the axioms above, the disjunctive permission value satisfies all axioms except structural monotonicity.[6] It satisfies a weaker monotonicity requiring the inequality only if player $j \in N$ dominates player $i \in N$ *completely* in the sense that all directed (permission) paths from a top-player in $T(D)$ to player i contain player j. We denote the set of players that completely dominate player i by $\overline{P}_D(i)$, i.e.,

$$\overline{P}_D(i) = \left\{ j \in \widehat{P}_D(i) \;\middle|\; \begin{array}{c} j \in \{h_1, \ldots, h_{t-1}\} \text{ for every sequence of nodes } h_1, \ldots, h_t \\ \text{such that } h_1 \in T(D),\ h_k \in P_D(h_{k+1}) \text{ for} \\ k \in \{1, \ldots, t-1\}, \text{ and } h_t = i \end{array} \right\}. \tag{2.4}$$

We also define $\overline{P}_D^{-1}(i) = \{j \in \widehat{P}_D^{-1}(i) : i \in \overline{P}_D(j)\}$.

Weak structural monotonicity For every $(N,v,D) \in \mathcal{GP}$ with $(N,v) \in \mathcal{G}_M$, if $i \in N$ and $j \in \overline{P}_D^{-1}(i)$ then $f_i(N,v,D) \ge f_j(N,v,D)$.

Further, the disjunctive permission value satisfies *disjunctive fairness* which states that deleting the arc between two players h and $j \in P_D^{-1}(h)$ (with $|P_D(j)| \ge 2$) changes the payoffs of players h and j by the same amount. Moreover, also the payoffs of all players i that completely dominate player h change by this same amount. The conjunctive permission value does not satisfy this disjunctive fairness. However, it satisfies the alternative *conjunctive fairness* which states that deleting the arc between two players h and $j \in P_D^{-1}(h)$ changes the payoffs of player j and any *other* predecessor $k \in P_D(j) \setminus \{h\}$ of j by the same amount. Moreover, also the payoffs of all players that completely dominate the other predecessor k change by this same amount.

For acyclic $(N,D) \in \mathcal{D}_A$, $h \in N$ and $j \in P_D^{-1}(h)$, we denote the permission structure that is left after deleting the arc between h and j by

$$D_{-(h,j)} = D \setminus \{(h,j)\}.$$

Disjunctive fairness For every $(N,v,D) \in \mathcal{GP}$ with $(N,D) \in \mathcal{D}_A$, if $h \in N$ and $j \in P_D^{-1}(h)$ with $|P_D(j)| \ge 2$, then $f_j(N,v,D) - f_j(N,v,D_{-(h,j)}) = f_i(N,v,D) - f_i(N,v,D_{-(h,j)})$ for all $i \in \{h\} \cup \overline{P}_D(h)$.

Conjunctive fairness For every $(N,v,D) \in \mathcal{GP}$ with $(N,D) \in \mathcal{D}_A$, if $h,j,k \in N$ are such that $h \ne k$ and $h,k \in P_D(j)$, then $f_j(N,v,D) - f_j(N,v,D_{-(h,j)}) = f_i(N,v,D) - f_i(N,v,D_{-(h,j)})$ for all $i \in \{k\} \cup \overline{P}_D(k)$.

---

[5] We refer to van den Brink and Gilles (1996), van den Brink (1997, 1999) for a discussion of these properties.

[6] The axioms that are defined before for the class of all games with a permission structure can be straightforwardly defined on any subclass of games with a permission structure.

Theorem 2.2. [7]

(i) *(van den Brink, 1997) A solution f on the class of games with an acyclic permission structure is equal to the disjunctive permission value φ^d if and only if it satisfies efficiency, additivity, the inessential player property, the necessary player property, weak structural monotonicity and disjunctive fairness.*
(ii) *(van den Brink, 1999) A solution f on the class of games with an acyclic permission structure is equal to the conjunctive permission value φ^c if and only if it satisfies efficiency, additivity, the inessential player property, the necessary player property, weak structural monotonicity and conjunctive fairness.*

2.3. Polluted river problems

Consider the *cost sharing problem on a river network*, shortly called *polluted river problem*, on rivers with multiple springs (sink tree structures) as introduced by Dong et al. (2012), generalizing Ni and Wang (2007). Such a polluted river problem is given by a triple (N, D, c), where $N \subset \mathbb{N}$ is a finite set of agents located along a river, $D \subset N \times N$ is a sink tree that represents the river structure, and $c \in \mathbb{R}^N_+$ is an $|N|$-dimensional cost vector.[8] The river structure D is such that the river water flows along the arcs in D with $(i, j) \in D$ when the river water flows from agent i to its downstream neighbor j. So, the arcs in D are the river segments. The sink is denoted by $L \in N$. From here the river flows into a sea or lake. The cost vector $c \in \mathbb{R}^N_+$ is such that c_i is the cost of cleaning the river segment between agent i and its unique downstream neighbor. For the sink, c_L is the cost of cleaning the river before it flows into the sea or lake. We denote by $\mathcal{R}$ the class of all polluted river problems (N, D, c). Note that the agents in $P_D(i)$ are the upstream neighbors, and $P^{-1}_D(i)$ consists of the unique downstream neighbor of $i \in N$ in the river structure (N, D), where $|P^{-1}_D(i)| = 1$ for all $i \neq L$.

A *cost allocation* for a polluted river problem $(N, D, c) \in \mathcal{R}$ is a vector $y \in \mathbb{R}^N_+$, where y_i is the cost to be paid by agent $i \in N$ in the total joint cleaning cost of the river $\sum_{i\in N} c_i$. A *cost sharing method* g is a mapping that assigns a cost allocation $g(N, D, c) \in \mathbb{R}^N_+$ to every polluted river problem (N, D, c).

The following three cost sharing methods are introduced and axiomatized by Dong et al. (2012). First, the *Local Responsibility Sharing method*, shortly LRS method, assigns to every agent its own cost, and thus is given by

$$g^{LRS}_i(N, D, c) = c_i \text{ for all } i \in N. \tag{2.5}$$

The *Upstream Equal Sharing method*, shortly UES method, equally shares the cost of cleaning a certain river segment over all agents that are located upstream of that segment, and thus is given by

$$g^{UES}_i(N, D, c) = \sum_{j\in\{i\}\cup\widehat{P}^{-1}_D(i)} \frac{c_j}{\left|\{j\} \cup \widehat{P}_D(j)\right|} \text{ for all } i \in N. \tag{2.6}$$

Finally, the *Downstream Equal Sharing method*, shortly DES method, equally shares the cost of a certain river segment over all agents that are located downstream of that segment and thus is given by

$$g^{DES}_i(N, D, c) = \sum_{j\in\{i\}\cup\widehat{P}_D(i)} \frac{c_j}{\left|\{j\} \cup \widehat{P}^{-1}_D(j)\right|} \text{ for all } i \in N.$$

Dong et al. (2012) also associate three TU-games to polluted river problems $(N, D, c) \in \mathcal{R}$. The first one is the (additive) *stand-alone game* $L^{sa}_{(N,D,c)}$ given by[9]

$$L^{sa}_{(N,D,c)}(S) = \sum_{i\in S} c_i \quad \text{for all } S \subseteq N. \tag{2.7}$$

The second is the *Upstream-oriented game* $L^U_{(N,D,c)}$ given by

$$L^U_{(N,D,c)}(S) = \sum_{i\in S\cup\widehat{P}^{-1}_D(S)} c_i \quad \text{for all } S \subseteq N. \tag{2.8}$$

The third is the *Downstream-oriented game* $L^D_{(N,D,c)}$ given by

$$L^D_{(N,D,c)}(S) = \sum_{i\in S\cup\widehat{P}_D(S)} c_i \quad \text{for all } S \subseteq N.$$

[7] In the mentioned articles, these axiomatizations are shown for games with an acyclic and quasi-strongly connected permission structure. A digraph D is *quasi-strongly connected* if there exists an $i \in N$ such that $\widehat{P}^{-1}_D(i) = N \setminus \{i\}$. These results can straightforwardly be extended to games with an acyclic permission structure.

[8] We remark that our notation is slightly different from that of Dong et al. (2012) but the models are equivalent.

[9] We take the summation over the empty set to be equal to 0.

They show that the LRS, UES and DES methods can be obtained by applying the Shapley value to the stand-alone, Upstream-oriented, respectively Downstream-oriented game.

Further, Dong et al. (2012) provide axiomatizations using the following axioms, generalizing those of Ni and Wang (2007).

Efficiency For every $(N, D, c) \in \mathcal{R}$, it holds that $\sum_{i \in N} g_i(N, D, c) = \sum_{i \in N} c_i$.

Additivity For every $(N, D, c), (N, D, c') \in \mathcal{R}$, we have $g(N, D, c + c') = g(N, D, c) + g(N, D, c')$.

Independence of Irrelevant Costs For every $(N, D, c) \in \mathcal{R}$, and $i, j \in N$ such that $j \in N \setminus (\widehat{P}_D(i) \cup \{i\} \cup \widehat{P}_D^{-1}(i))$, we have that $g_j(N, D, c) = 0$ whenever $c_h = 0$ for all $h \in N \setminus \{i\}$.

Upstream Symmetry For every $(N, D, c) \in \mathcal{R}$ and $i \in N$, it holds that $g_j(N, D, c) = g_k(N, D, c)$ for all $j, k \in \{i\} \cup \widehat{P}_D(i)$, whenever $c_h = 0$ for all $h \in N \setminus \{i\}$.

Independence of Upstream Costs For every $(N, D, c), (N, D, c') \in \mathcal{R}$ and $i \in N$ such that $c_h = c'_h$ for all $h \in \widehat{P}_D^{-1}(i)$, we have that $g_j(N, D, c) = g_j(N, D, c')$ for all $j \in \widehat{P}_D^{-1}(i)$.

Downstream Symmetry For every $(N, D, c) \in \mathcal{R}$ and $i \in N$, it holds that $g_j(N, D, c) = g_k(N, D, c)$ for all $j, k \in \{i\} \cup \widehat{P}_D^{-1}(i)$, whenever $c_h = 0$ for all $h \in N \setminus \{i\}$.

Independence of Downstream Costs For every $(N, D, c), (N, D, c') \in \mathcal{R}$ and $i \in N$ such that $c_h = c'_h$ for all $h \in \widehat{P}_D(i)$, we have that $g_j(N, D, c) = g_j(N, D, c')$ for all $j \in \widehat{P}_D(i)$.

Efficiency and additivity are standard axioms. Independence of irrelevant costs means that, if there is only one country with a positive cleaning cost, then the only countries that (possibly) contribute to the cleaning cost are this country and its up- and downstream countries. Upstream symmetry means that in case there is only one country with a positive cost, then this country contributes the same as each of its upstream countries (who therefore all pay the same). Independence of upstream costs implies that the contribution of a country does not depend on changes in the cleaning costs at its upstream countries. Similar interpretations can be given to downstream symmetry and independence of downstream costs.[10]

Theorem 2.3 *(Dong et al., 2012).* [11]

(i) *The UES method is the unique cost sharing method satisfying efficiency, additivity, independence of irrelevant costs, upstream symmetry and independence of upstream costs.*
(ii) *The DES method is the unique cost sharing method satisfying efficiency, additivity, independence of irrelevant costs, downstream symmetry and independence of downstream costs.*

3. The UES method and the conjunctive permission value

3.1. An axiomatization

In van den Brink et al. (2014a) it is mentioned that, in case the river has a single spring (as in Ni and Wang, 2007), the Upstream-oriented game $L^U_{(N,D,c)}$ (see (2.8)) associated to a polluted river problem (N, D, c) equals the dual game of the conjunctive restricted game of the game with permission structure $(N, L^{sa}_{(N,D,c)}, D)$ where $L^{sa}_{(N,D,c)}$ is the stand-alone game (see (2.7)) and D is the permission structure associated to the river structure with the arcs oriented from upstream to downstream. This can be extended to rivers with multiple springs.

Proposition 3.1. *For every polluted river problem $(N, D, c) \in \mathcal{R}$, the Upstream-oriented game $L^U_{(N,D,c)}$ is equivalent to the dual game of $r^c_{L^{sa}_{(N,D,c)}, D}$.*[12]

Proof. Recall that the dual game of a game v, denoted by $\widetilde{v}$, on player set N is given by

$$\widetilde{v}(S) = v(N) - v(N \setminus S) \quad \text{for each } S \subseteq N.$$

[10] For an extensive discussion on the solutions and relating these axioms to water allocation principles, we refer to Ni and Wang (2007) and Dong et al. (2012).

[11] Besides these axiomatizations, Dong et al. (2012) axiomatize the LRS method by Efficiency, Additivity and No Blind Cost, the last axiom requiring that for every $(N, D, c) \in \mathcal{R}$ and $i \in N$ such that $c_i = 0$, we have $g_i(N, D, c) = 0$.

[12] This proposition holds under the more general condition that D is acyclic. The proof here does not require that D is a sink tree.

From this definition, one has $\widetilde{r}^c_{L^{sa}_{(N,D,c)},D}(\emptyset)=0$, which coincides with $L^U_{(N,D,c)}(\emptyset)=0$. For every non-empty subset $S\subseteq N$, define $\sigma^c_D(S)=\bigcup_{\{T\in\Phi^c_D:T\subseteq S\}}T=\{i\in S:\widehat{P}_D(i)\subseteq S\}$ being the largest conjunctive feasible subset of S. Thus $r^c_{v,D}(S)=v(\sigma^c_D(S))$ for all $S\subseteq N$. Since, for every $S\subseteq N$, $\sigma^c_D(N\setminus S)=\{i\in N\setminus S:\widehat{P}_D(i)\subseteq N\setminus S\}=\{i\in N\setminus S:\widehat{P}_D(i)\cap S=\emptyset\}=(N\setminus S)\setminus\widehat{P}^{-1}_D(S)$, we have

$$\begin{aligned}\widetilde{r}^c_{L^{sa}_{(N,D,c)},D}(S)&=r^c_{L^{sa}_{(N,D,c)},D}(N)-r^c_{L^{sa}_{(N,D,c)},D}(N\setminus S)\\&=\sum_{i\in N}c_i-L^{sa}_{(N,D,c)}(\sigma^c_D(N\setminus S))\quad=\sum_{i\in N}c_i-L^{sa}_{(N,D,c)}((N\setminus S)\setminus\widehat{P}^{-1}_D(S))\\&=\sum_{i\in N}c_i-\sum_{i\in(N\setminus S)\setminus\widehat{P}^{-1}_D(S)}c_i\quad=\sum_{i\in S}c_i+\sum_{i\in(N\setminus S)\cap\widehat{P}^{-1}_D(S)}c_i\quad=\sum_{i\in S\cup\widehat{P}^{-1}_D(S)}c_i\\&=L^U_{(N,D,c)}(S).\qquad\square\end{aligned}$$

Since the conjunctive permission value of a game with a permission structure is obtained as the Shapley value of the corresponding conjunctive restricted game, and the Shapley value of a game is equal to the Shapley value of its dual game, i.e. $Sh(v)=Sh(\widetilde{v})$ for all $(N,v)\in\mathcal{G}$ (see Kalai and Samet, 1987), it follows with Proposition 3.1 that the UES method can be obtained by applying the conjunctive permission value to the game with permission structure $(N,L^{sa}_{(N,D,c)},D)$.

Corollary 3.2. *Let $(N,D,c)\in\mathcal{R}$ be a polluted river problem. Then*

$$g^{UES}(N,D,c)=\varphi^c(N,L^{sa}_{(N,D,c)},D).$$

Since Corollary 3.2 shows that the UES method can be obtained by applying the conjunctive permission value to the stand-alone game on the up-downstream oriented (river) permission structure D, we can verify the implication of the axioms underlying the conjunctive permission value for polluted river problems mentioned in Section 2, and investigate if axioms that characterize the conjunctive permission value also give uniqueness on the class of Upstream-oriented games $\mathcal{GPR}=\{(N,v,D)\in\mathcal{GP}:v=L^{sa}_{(N,D,c)}$ for some $(N,D,c)\in\mathcal{R}\}=\{(N,v,D)\in\mathcal{GP}:v$ is inessential with $v(\{i\})\geq 0$ for all $i\in N$ and D is a sink tree $\}\subset\mathcal{GP}$. Instead of considering this class of games with a permission structure, we directly interpret and apply the axioms in terms of polluted river problems.[13] It turns out that these axioms do not only provide uniqueness, but also are a good reflection of established water allocation principles from International Water Law.

To show equivalence between properties of solutions for games with a permission structure and cost sharing methods for polluted river problems, we say that a cost sharing method g is an *Upstream-oriented game method* if there is a solution f for games with a permission structure such that $g(N,D,c)=f(N,L^{sa}_{(N,D,c)},D)$ for all $(N,D,c)\in\mathcal{R}$. Now, we can first state that efficiency for permission values on the class $\mathcal{GPR}$ is equivalent to efficiency for polluted river cost sharing methods in the sense that cost sharing method g given by $g(N,D,c)=f(N,L^{sa}_{(N,D,c)},D)$ satisfies efficiency on $\mathcal{R}$ if and only if solution f satisfies efficiency on $\mathcal{GPR}$. In this sense also additivity for permission values on the class $\mathcal{GPR}$ is equivalent to additivity for polluted river cost sharing methods. The obvious proofs are omitted.

Next, we interpret the other axioms of Theorem 2.1. Since an agent is an inessential player in game with permission structure $(N,L^{sa}_{(N,D,c)},D)$ for some $(N,D,c)\in\mathcal{R}$, if and only if its own cost as well as the cost of all its subordinates is zero, the inessential player property for polluted river games with a permission structure is equivalent to requiring zero contributions for such agents.

Inessential agent property For every $(N,D,c)\in\mathcal{R}$ and $i\in N$ such that $c_j=0$ for all $j\in\widehat{P}^{-1}_D(i)\cup\{i\}$, it holds that $g_i(N,D,c)=0$.

The inessential agent property is stronger than independence of irrelevant costs since it also states requirements for the payoffs in polluted river problems where more than one agent has a positive cleaning cost. Moreover, independence of irrelevant costs only considers cases where costs are zero for an agent, all its superiors and all its subordinates, while the inessential agent property can apply when superiors have a positive cost. We relate the axioms to water allocation principles at the end of this subsection.

Proposition 3.3. *Every cost sharing method that satisfies the inessential agent property also satisfies independence of irrelevant costs.*

[13] Note that there is a one-to-one correspondence between games with permission structure (N,v,D) with v an inessential game and D a sink tree, and polluted river problems. Above we saw that every polluted river problem (N,D,c) yields a game with a permission structure (N,v,D) with the permission structure D and the inessential game v determined by c. On the other hand, given an inessential game v with a sink tree permission structure D, the corresponding polluted river problem is determined by the permission structure D with costs equal to $c_i=v(\{i\})$ for all $i\in N$.

Proof. Suppose that cost sharing method g satisfies the inessential agent property, and let river problem $(N, D, c) \in \mathcal{R}$ be such that there is an $i \in N$ with $c_h = 0$ for all $h \in N \setminus \{i\}$. For $j \in N \setminus (\widehat{P}_D(i) \cup \{i\} \cup \widehat{P}_D^{-1}(i))$, we have that $c_k = 0$ for all $k \in \widehat{P}_D^{-1}(j) \cup \{j\}$, and thus $g_j(N, D, c) = 0$ by the inessential agent property. Thus, g satisfies independence of irrelevant costs. □

Since an agent is a necessary player in a game with permission structure $(N, L^{sa}_{(N,D,c)}, D)$ for some $(N, D, c) \in \mathcal{R}$ if and only if the costs of all other agents are zero, and stand-alone games are monotone, the necessary agent property for polluted river games with a permission structure is equivalent to requiring that such an agent contributes at least as much as any other agent.

Necessary agent property For every $(N, D, c) \in \mathcal{R}$ and $i \in N$ with $c_j = 0$ for all $j \in N \setminus \{i\}$, it holds that $g_i(N, D, c) \geq g_j(N, D, c)$ for all $j \in N \setminus \{i\}$.

Finally, structural monotonicity for permission values is equivalent to requiring that upstream agents contribute at least as much as downstream agents.

Structural monotonicity For every $(N, D, c) \in \mathcal{R}$ and $i, j \in N$ with $i \in P_D(j)$, it holds that $g_i(N, D, c) \geq g_j(N, D, c)$.

Note that structural monotonicity implies that $g_i(N, D, c) \geq g_j(N, D, c)$ for all $i \in \widehat{P}_D(j)$. The necessary agent property and structural monotonicity together are stronger than upstream symmetry.

Proposition 3.4. *Every cost sharing method that satisfies the necessary agent property and structural monotonicity also satisfies upstream symmetry.*[14]

Proof. Suppose that cost sharing method g satisfies the necessary agent property and structural monotonicity, and let polluted river problem $(N, D, c) \in \mathcal{R}$ be such that there is an $i \in N$ with $c_h = 0$ for all $h \in N \setminus \{i\}$. The necessary agent property implies that $g_i(N, D, c) \geq g_j(N, D, c)$ for all $j \in \widehat{P}_D(i)$. Structural monotonicity implies that $g_i(N, D, c) \leq g_j(N, D, c)$ for all $j \in \widehat{P}_D(i)$. Together these imply that $g_i(N, D, c) = g_j(N, D, c)$ for all $j \in \widehat{P}_D(i)$, and thus g satisfies upstream symmetry. □

It turns out that replacing independence of irrelevant costs, upstream symmetry and independence of upstream costs in Theorem 2.3 (i) by the inessential agent property, the necessary agent property and structural monotonicity characterizes the UES method.

Theorem 3.5. *The UES method is the unique cost sharing method that satisfies efficiency, additivity, the inessential agent property, the necessary agent property and structural monotonicity.*

Proof. It is straightforward from its definition (2.6) that the UES method satisfies the five axioms. To show uniqueness, suppose that cost sharing method g satisfies the five axioms, and consider polluted river problem $(N, D, c) \in \mathcal{R}$. For every $i \in N$, define $c^i \in \mathbb{R}^N_+$ by $c^i_i = c_i$ and $c^i_j = 0$ for all $j \in N \setminus \{i\}$. The inessential agent property implies that $g_j(N, D, c^i) = 0$ for all $j \in N \setminus (\{i\} \cup \widehat{P}_D(i))$. By Proposition 3.4, g satisfies upstream symmetry, and thus $g_i(N, D, c^i) = g_j(N, D, c^i)$ for all $j \in \widehat{P}_D(i)$. Efficiency then determines that $g_i(N, D, c^i) = g_j(N, D, c^i) = c_i \big/ \big(|\widehat{P}_D(\{i\})| + 1\big)$ for all $j \in \widehat{P}_D(i)$, which equals the payoffs assigned by the UES method. Finally, additivity determines the payoffs according to the UES method for any polluted river problem $(N, D, c) \in \mathcal{R}$ since $c = \sum_{i \in N} c^i$. □

Logical independence of the five axioms in Theorem 3.5 is shown in Appendix B.

Since the axioms of Theorem 3.5 are direct applications of the axioms for the conjunctive permission value in van den Brink and Gilles (1996), this also shows that these axioms characterize the conjunctive permission value on the subclass of games with a permission structure $\mathcal{GPR}$ that are obtained from polluted river problems, i.e. nonnegative additive games on sink trees. Moreover, we have put the UES method for polluted river problems in the broader context of games with a permission structure.

Comparing the axioms of Theorems 2.3 and 3.5, we replaced upstream symmetry (which is a rather strong equity principle), by the necessary agent property and structural monotonicity, and thus, we split upstream symmetry in two properties that each reflect a different allocation principle. By further strengthening independence of irrelevant costs to the inessential agent property, a main advantage of Theorem 3.5 is that we do not need the rather strong independence of upstream costs.

Turning to water allocation principles, water resources sharing and water pollution cost sharing methods have in common that they provide rules for upstream and downstream agents to reach agreement on the allocation or cleaning of river water. The axioms of Theorem 3.5 reflect such water allocation principles. *Efficiency* and *additivity* are discussed by

[14] Neither the necessary agent property nor structural monotonicity on its own implies upstream symmetry, and upstream symmetry implies neither the necessary agent property nor structural monotonicity. We show this in Appendix A.

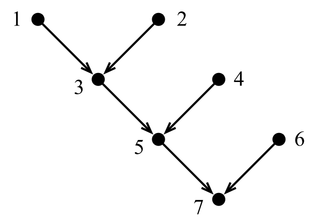

Fig. 1. A river with 7 agents.

Ni and Wang (2007) and Dong et al. (2012). In the introduction, we mentioned the tension between the ATS and TIBS principles, although TIBS can be interpreted in several ways. For the allocation of clean river water, Ambec and Sprumont (2002) take the Unlimited Territorial Integrity (UTI) interpretation saying that a state has the right to demand the natural flow of an international watercourse into its territory that is undiminished by its upstream states (stated in the rules of the Helsinki Convention on water rights of the International Law Association, 1966). For polluted river problems, Ni and Wang (2007) and Dong et al. (2012) interpret UTI as a Downstream Responsibility (DR) principle which says that 'an agent is responsible for the cost of cleaning her own link and partially responsible for the costs of all her downstream links'. This principle is reflected by *structural monotonicity* which requires higher responsibility to an upstream agent compared to its downstream ones. Dong et al. (2012) motivate independence of irrelevant costs, which requires that in case there is only one agent with positive cost, all agents that are neither upstream nor downstream of the agent itself contribute zero, purely as an equity axiom. Our stronger *inessential agent property* requires that in this case the agents that are not upstream of this agent nor the agent itself, pay zero contribution. Therefore, this is not a pure equity axiom, but also reflects the Downstream Responsibility interpretation of UTI in the sense that, compared to independence of irrelevant costs, it requires that the cost of this agent is paid by this agent and its upstream agents. Finally, the *necessary agent property* can be seen as a weaker version of Local Responsibility as introduced by Ni and Wang (2007) and Dong et al. (2012) as an interpretation of *Absolute Territorial Sovereignty (ATS)* (discussed in the introduction). This principle emphasizes the local right. It implies that an agent has absolute responsibility over the cost at its local river segment. Whereas Ni and Wang (2007) and Dong et al. (2012) use it to motivate the Local Responsibility Sharing (LRS) method (see (2.5)), the necessary agent property weakens it in two ways. First, it only requires local responsibility for the case where there is only one agent with a positive cost. Second, although it does not require the local agent to be fully responsible for its own cost, it does require that the local agent shares at least as much as any other agent in its own cost. Or, in other words, the local agent is always at least as much responsible for the costs generated within its river basin than any other agent, but not necessarily fully responsible.[15]

3.2. Externality fairness: a new axiomatization

By considering a polluted river problem as a game with a permission structure, we can also obtain new characterizations of the UES method, applying the idea behind conjunctive fairness (see Theorem 2.2.(ii)). Suppose that an agent with all its upstream agents stop being part of the pollution cleaning agreement. If we model this by deleting the link between this agent and its downstream neighbor, then this results in two different river structures that act as if not connected to each other. Although the river structure itself does not change, the *cooperation structure*, which initially is the same as the river structure, might 'break up' in different components. Thus, the cooperation structure which reflects the participated agents in the agreement, is now a subgraph of the river structure.

Note that in conjunctive fairness for games with a permission structure, deleting an arc (i, j) means that j does not need permission anymore from i to cooperate with other players. In the polluted river problem, when i stops participation in an agreement with $j \in P_D^{-1}(i)$, i and all its superiors (upstream agents) will make a new agreement on their own, and similarly for j with the rest of the agents. This brings up the axiom of *externality fairness*. Suppose that the sub-river consisting of i and all its superiors retreat from the agreement and only pay their own cost and do not contribute anymore in the cleaning cost of the others, in particular not of j and its subordinates (downstream agents). Of course, then those other agents will not contribute to the cleaning cost of i and its superiors, and this complement should pay its own cost. Externality fairness states that in this case the change (increase) of the contribution of j in the cost of its component (in the new cooperation structure) should be equal to the change in the contribution of any of its other upstream neighbors. So, the refusal of an upstream neighbor of j to contribute to the cleaning cost in the river component with j affects the contributions of the other upstream neighbors of j by the same amount as j.

Example 3.6. Consider a polluted river problem on the river structure depicted in Fig. 1. Externality fairness implies that when, for example, agent 3 (and its upstream agents 1 and 2) stop the cost allocation agreement with its downstream neighbor 5 and the other agents, then the effect in the contribution of agent 5 is the same as for agent 4.

[15] Full responsibility is expressed, for example, by the axiom of No Blind Cost (see Footnote 9) which with efficiency and additivity characterizes the Local Responsibility Sharing method, see Ni and Wang (2007) for single spring rivers and Dong et al. (2012) for sink tree rivers.

Before formally stating the axiom, we introduce some notation. For river structure D, let $K_{ij}^{j}(D)$, $(i,j) \in D$, be the component containing j that is created after the deletion of the arc (i,j), i.e. $K_{ij}^{j}(D) = N \setminus (\{i\} \cup \widehat{P}_D(i))$. To simplify, we denote $K_{ij}^{j}(D)$ by K_{ij}^{j}. Note that L remains the sink in the polluted river problem $(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}})$, where $D|_{K_{ij}^{j}} = \{(h,k) \in D : \{h,k\} \subseteq K_{ij}^{j}\}$ is the river structure restricted to K_{ij}^{j} (note that this is again a sink tree), and $c|_{K_{ij}^{j}}$ is the projection of the cost vector c on K_{ij}^{j}.

Externality fairness For every polluted river problem $(N,D,c) \in \mathcal{R}$ and $i,j \in N$ with $(i,j) \in D$, it holds that

$$g_j(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}}) - g_j(N,D,c) = g_h(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}}) - g_h(N,D,c)$$

for every $h \in P_D(j) \setminus \{i\}$.

Note that, besides a difference in interpretation, another difference with conjunctive fairness is that we only require equal change in payoffs for j and its upstream neighbors, while conjunctive fairness also requires this for the 'complete superiors' of the upstream neighbors of j. The sink tree structure of the river allows that we only need to consider the direct predecessors of j.

Using externality fairness, for sink trees we can weaken structural monotonicity by requiring it only for an agent and its unique upstream neighbor.

Weak structural monotonicity For every polluted river problem $(N,D,c) \in \mathcal{R}$ and every $j \in N$, if $P_D(j) = \{i\}$, then $g_i(N,D,c) \geq g_j(N,D,c)$.

For polluted river problems, this is a considerable weakening of structural monotonicity since it only requires monotonicity with respect to an agent and its upstream neighbor in case it is its unique upstream neighbor. It turns out that when a cost sharing method satisfies externality fairness, then weak structural monotonicity implies structural monotonicity.

Proposition 3.7. *Every cost sharing method that satisfies externality fairness and weak structural monotonicity also satisfies structural monotonicity.*

Proof. Suppose that cost sharing method g satisfies externality fairness and weak structural monotonicity, and consider polluted river problem $(N,D,c) \in \mathcal{R}$. It is obvious that the claim holds for line-rivers, i.e. with $|T(D)| = 1$, since in that case weak structural monotonicity is equivalent to structural monotonicity. We show that the claim also holds for general sink tree river structures by induction on the number of springs $|T(D)|$. Assume that the claim holds for all rivers with $|T(D)| \leq m$ for some $m \geq 1$. Now for rivers with $|T(D)| = m+1$, for any $j \in N$, if $|P_D(j)| = 1$, weak structural monotonicity implies that $g_i(N,D,c) \geq g_j(N,D,c)$ for $i \in P_D(j)$. If $|P_D(j)| > 1$, then for any $i \in P_D(j)$, externality fairness implies that

$$g_j(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}}) - g_j(N,D,c) = g_h(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}}) - g_h(N,D,c) \tag{3.9}$$

for all $h \in P_D(j) \setminus \{i\}$. Note that the number of springs of $D|_{K_{ij}^{j}}$ is less than $m+1$. From the induction hypothesis we have $g_h(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}}) \geq g_j(K_{ij}^{j}, D|_{K_{ij}^{j}}, c|_{K_{ij}^{j}})$, which with (3.9) implies that $g_h(N,D,c) \geq g_j(N,D,c)$. Thus g satisfies structural monotonicity. $\square$

Note that structural monotonicity implies weak structural monotonicity, but does not imply externality fairness. This is illustrated by, for example, the method $g^{\overline{UES}}$ defined in Appendix B.

Theorem 3.8. *The UES method is the unique cost sharing method that satisfies efficiency, additivity, the inessential agent property, the necessary agent property, weak structural monotonicity and externality fairness.*

Proof. It is straightforward to verify that the UES method satisfies the first five axioms. To show externality fairness, for any polluted river problem $(N,D,c) \in \mathcal{R}$ and any $i,j,h \in N$ such that $i,h \in P_D(j)$, $i \neq h$, denoting $D' = D|_{K_{ij}^{j}}$, it holds that

$$\begin{aligned} & g_h^{UES}(K_{ij}^{j}, D', c|_{K_{ij}^{j}}) - g_j^{UES}(K_{ij}^{j}, D', c|_{K_{ij}^{j}}) \\ &= \sum_{k \in \{h\} \cup \widehat{P}_{D'}^{-1}(h)} \frac{c_k}{\left|\{k\} \cup \widehat{P}_{D'}(k)\right|} - \sum_{k \in \{j\} \cup \widehat{P}_{D'}^{-1}(j)} \frac{c_k}{\left|\{k\} \cup \widehat{P}_{D'}(k)\right|} \\ &= \frac{c_h}{\left|\{h\} \cup \widehat{P}_{D'}(h)\right|}, \end{aligned}$$

and

$$g_h^{UES}(N,D,c) - g_j^{UES}(N,D,c) = \sum_{k\in\{h\}\cup\widehat{P}_D^{-1}(h)} \frac{c_k}{\left|\{k\}\cup\widehat{P}_D(k)\right|} - \sum_{k\in\{j\}\cup\widehat{P}_D^{-1}(j)} \frac{c_k}{\left|\{k\}\cup\widehat{P}_D(k)\right|}$$
$$= \frac{c_h}{\left|\{h\}\cup\widehat{P}_D(h)\right|}$$

Since the number of superiors of h in D is equal to that in $D' = D|_{K_{ij}^j}$, one has

$$g_h^{UES}(K_{ij}^j, D|_{K_{ij}^j}, c|_{K_{ij}^j}) - g_j^{UES}(K_{ij}^j, D|_{K_{ij}^j}, c|_{K_{ij}^j}) = g_h^{UES}(N,D,c) - g_j^{UES}(N,D,c),$$

implying that the UES method satisfies externality fairness.

Uniqueness follows from Proposition 3.7 and Theorem 3.5. □

Logical independence of the six axioms in Theorem 3.8 is again shown in Appendix B.

Compared to the previous section we replaced structural monotonicity by weak structural monotonicity and externality fairness. Considering water allocation principles resulting from international water resources sharing disputes, similar to structural monotonicity, weak structural monotonicity reflects the Downstream Responsibility interpretation of UTI but in a weaker form. Additionally, externality fairness requires that when an upstream agent stops the agreement with a downstream neighbor, the responsibility of the additional contribution to be made by the downstream agent and that to be made by any of its other upstream neighbors are equal. Similar as Dong et al. (2012)'s upstream symmetry, this combines equity with the Downstream Responsibility interpretation of UTI in the sense that it equalizes the changes of payoffs of different agents in case the (polluted river) situation changes in an equivalent way from the perspective of the responsibilities of these agents.

4. The ULS method and the disjunctive permission value

Considering polluted river problems as games with a permission structure, we can define a new cost sharing method for polluted river problems by applying the disjunctive permission value to every polluted river problem. For sink trees, the conjunctive and disjunctive permission value differ except for directed line-graphs, i.e. single-spring rivers. Therefore, for all rivers with a sink tree structure with at least two springs, applying the disjunctive permission value φ^d yields a new allocation method for polluted river problems.

Definition 4.1. The *Upstream Limited Sharing* method (ULS method) is given by

$$g^{ULS}(N,D,c) = \varphi^d(N, L^{sa}_{(N,D,c)}, D) \text{ for every } (N,D,c)\in\mathcal{R} \text{ with } \varphi^d \text{ given by (2.3).}$$

The idea behind this ULS method is that agents who are predecessor, but not the only predecessor, of a downstream agent feel less responsible for cleaning the river at their downstream agent than according to the UES method, but still take some responsibility. Consider, for example, the river (N,D,c) with $N=\{1,2,3\}$, $D=\{(1,3),(2,3)\}$ (and thus $L=3$) and $c=(c_1,c_2,c_3)=(0,0,c_3)$ with $c_3>0$, see Fig. 2. According to the UES method, the cost c_3 is equally shared by the agents 1, 2 and 3, i.e. $g^{UES}(N,D,c)=(c_3/3, c_3/3, c_3/3)$. According to the ULS method the cost shares are $g^{ULS}(N,D,c) = (c_3/6,\ c_3/6,\ 2c_3/3)$ which are obtained as the Shapley value of the restricted game $r^d_{L^{sa}_{(N,D,c)},D}$ given by $r^d_{L^{sa}_{(N,D,c)},D}(S)=c_3$ if $S\in\{\{1,3\},\{2,3\},\{1,2,3\}\}$, and $r^d_{L^{sa}_{(N,D,c)},D}(S)=0$ otherwise. Agent 1 can argue that it is not responsible for the pollution at agent 3 (since it claims that the pollution comes from agent 2). Consequently, the contribution of agent 1, $c_3/6$, is less than when agents 1, 2 and 3 are held equally responsible for the pollution at agent 3 (as in the UES method where agent 1 contributes $c_3/3$). The same argument holds for agent 2, yielding a cost allocation where the upstream agents 1 and 2 pay less in the cleaning cost at 3 than in the UES method. Although agent 3 might argue that the pollution comes from 1 or 2, the uncertainty about which agent is responsible yields a smaller responsibility and contribution of the upstream neighbors 1 and 2. Still, agents 1 and 2 contribute, contrary to the Local Responsibility Sharing (LRS) method where their contribution to c_3 is zero. Therefore, the ULS method yields some kind of compromise between the UES method and LRS method in the sense that according to the LRS method agent 3 has to pay its cost fully with no contribution from other agents, while according to the UES method c_3 is equally shared among agent 3 and its upstream agents. According to the ULS method, the upstream agents 1 and 2 do contribute in the cleaning cost of agent 3, but less than agent 3.

Notice that in case of a linear river with a unique top and every other agent having exactly one upstream neighbor, for each non-top agent having all predecessors in a coalition is equivalent to having at least one predecessor in a coalition, and thus the sets of conjunctive feasible (Φ_D^c) and disjunctive feasible (Φ_D^d) coalitions coincide. Consequently, the conjunctive and disjunctive restricted games and permission values, and therefore the UES and ULS methods, coincide for the linear river case.

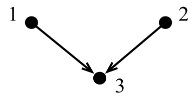

Fig. 2. A river with 3 agents.

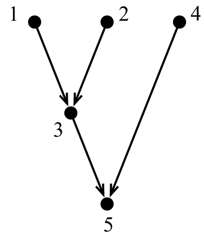

Fig. 3. A river with 5 agents.

Definition 4.1 is an indirect one in the sense that it is based on a disjunctive restricted game defined on another game generated from a polluted river problem. Here we provide an alternative direct definition which is more transparent than the two step definition given before and helps in evaluating the ULS solution.

Define the Limited Upstream-oriented coalition $Q(S)$ for $S \subseteq N$ as

$$Q(S) = \bigcap \left\{ F \;\middle|\; S \subseteq F \subseteq N, \emptyset \neq P_D(i) \subseteq F \Rightarrow i \in F \right\}. \tag{4.10}$$

The set $Q(S)$ contains two types of agents that are important with respect to the feasibility of agents in S. First, if i is a top agent, i.e. $i \in T(D)$, then $i \in S$ is necessary and sufficient to have $i \in Q(S)$. Second, if i is not a top agent, i.e. $i \notin T(D)$, then $i \in Q(S)$ if and only if $i \in S$ or $P_D(i) \subseteq Q(S)$. In other words, the set $Q(S)$ consists of all agents in S and all agents $i \in N \setminus S$ such that for every path $\{h_1, \ldots, h_t\}$ with $h_1 \in T(D)$, $h_k \in P_D(h_{k+1})$ for all $k \in \{1, \ldots, t-1\}$, $h_t = i$, it holds that $\{h_1, \ldots, h_t\} \cap S \neq \emptyset$. As an illustration, consider the river structure $D = \{(1,3), (2,3), (3,5), (4,5)\}$ on $N = \{1,2,3,4,5\}$, see Fig. 3. In this river structure, $Q(\{1\}) = \{1\}$ since for every agent other than agent 1, there is a path from a top to that agent that does not contain agent 1. For agent 3, we have $Q(\{3\}) = \{3\}$, and thus this does not contain its unique subordinate 5 since there is a path from top agent 4 to agent 5 that does not contain agent 3. Similar, $Q(\{4\}) = \{4\}$. But $Q(\{3,4\}) = \{3,4,5\}$ contains agent 5 because every path from a top to agent 5 contains either agent 3 (the paths $(1,3,5)$ and $(2,3,5)$) or agent 4 (the path $(4,5)$). By definition, $Q(T(D)) = Q(\{1,2,4\}) = \{1,2,3,4,5\}$ because every path to an agent should contain a top agent. Obviously, it holds that $Q(\emptyset) = \emptyset$ and $S \subseteq Q(S)$. The Limited Upstream-oriented game $L^{LU}_{(N,D,c)}$ associated to the polluted river problem $(N,D,c) \in \mathcal{R}$ is defined by

$$L^{LU}_{(N,D,c)}(S) = \sum_{i \in Q(S)} c_i \quad \text{for all } S \subseteq N. \tag{4.11}$$

It turns out that the Limited Upstream-oriented game $L^{LU}_{(N,D,c)}$ associated to a polluted river problem (N,D,c) equals the dual game of the disjunctive restricted game of the game with permission structure $(N, L^{sa}_{(N,D,c)}, D)$ of the stand-alone game $L^{sa}_{(N,D,c)}$ on the permission structure D associated to the river structure with the arcs oriented from upstream to downstream.

Proposition 4.2. *For every polluted river problem $(N,D,c) \in \mathcal{R}$, $L^{LU}_{(N,D,c)}$ is the dual game of $r^d_{L^{sa}_{(N,D,c)},D}$.*

Proof. Denote the dual game of $L^{LU}_{(N,D,c)}$ by $\widetilde{L}^{LU}_{(N,D,c)}$. Thus,

$$\widetilde{L}^{LU}_{(N,D,c)} = L^{LU}_{(N,D,c)}(N) - L^{LU}_{(N,D,c)}(N \setminus S) = \sum_{i \in N} c_i - \sum_{i \in Q(N \setminus S)} c_i = \sum_{i \notin Q(N \setminus S)} c_i.$$

From the definition of $Q(S)$ it holds that $i \notin Q(N \setminus S)$ if and only if

$$\begin{cases} i \notin N \setminus S & \text{if } i \in T(D), \\ i \notin N \setminus S \text{ and } \exists j \in P_D(i) \text{ such that } j \notin Q(N \setminus S) & \text{if } i \notin T(D). \end{cases}$$

Define $Q^*(S) := N \setminus Q(N \setminus S)$. The fact above can be rewritten as $i \in Q^*(S)$ if and only if

$$\begin{cases} i \in S & \text{if } i \in T(D), \\ i \in S \text{ and } \exists j \in P_D(i) \text{ such that } j \in Q^*(S) & \text{if } i \notin T(D). \end{cases}$$

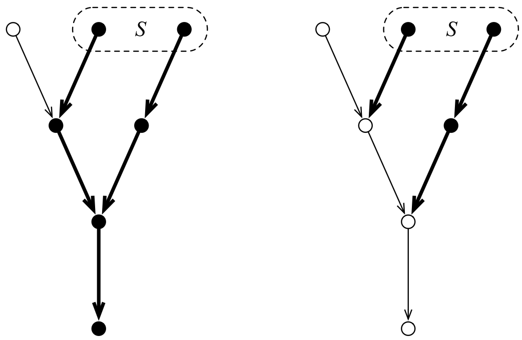

Fig. 4. A comparison between $S \cup \widehat{P}_D^{-1}(S)$ and $Q(S)$.

It is obvious that $Q^*(S) \subseteq S \subseteq Q(S)$. Therefore, it can be seen that $Q^*(S)$ is the largest disjunctive feasible subset of coalition S. Consequently, one has

$$\widetilde{L}^{LU}_{(N,D,c)} = \sum_{i \in Q^*(S)} c_i = r^d_{L^{sa}_{(N,D,c)},D}(S),$$

completing the proof. □

Since the Shapley value of a TU-game equals the Shapley value of its dual game, we have the following proposition by Definition 4.1 and Proposition 4.2.

Proposition 4.3.

$$g_i^{ULS}(N,D,c) = \sum_{S \subseteq N : i \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!} \left(\sum_{j \in Q(S)} c_j - \sum_{j \in Q(S \setminus \{i\})} c_j \right) \textit{ for all } i \in N. \tag{4.12}$$

Notice that Equation (4.12) provides a direct definition of the ULS method that is based only on the river network and the cost parameters (and does not refer to any game or the Shapley value).

Since the UES and ULS methods both are based on applying the Shapley value to an associated game, the difference between these methods can be explained by comparing the costs assigned to sets/regions of agents, i.e. the worths in the associated games. In the Upstream-oriented game (2.8), the worth of a coalition S is the sum of the costs of agents that are members of $S \cup \widehat{P}_D^{-1}(S)$ being region S together with its downstream region. If we imagine a dried up river and pour water into the sections belonging to S, then region $S \cup \widehat{P}_D^{-1}(S)$ can be seen as the sections where water flows through, see the left panel in Fig. 4.

For the Limited Upstream-oriented game (4.11), the worth of coalition S consists of the costs of agents that are members of $Q(S)$, being the region S and all agents i outside region S such that every path from a top agent to agent i contains an agent from region S. Now, we find the set $T_{-S}(D) = T(D) \setminus \widehat{P}_D(S)$, which are the springs of the river whose downstream regions do not go through S. Now, suppose that the springs in $T_{-S}(D)$ pollute the river on their territory such that the water is not useable anymore in all their downstream countries. In this way, these springs can block the access of useable water by the countries in $T_{-S}(D) \cup \widehat{P}_D^{-1}(T_{-S}(D))$. Then, the only countries that have access to clean water are those that belong to region $Q(S)$, see the right panel in Fig. 4. This also gives another expression for $Q(S)$ as

$$Q(S) = \left[S \cup \widehat{P}_D^{-1}(S) \right] \setminus \left[T_{-S}(D) \cup \widehat{P}_D^{-1}(T_{-S}(D)) \right].$$

4.1. Participation fairness: an axiomatization

If the cost sharing methods reflect the lower responsibility for upstream agents in case it is not sure where the pollution comes from, then the question becomes to what extent this uncertainty should be reflected in the cost sharing methods. Here disjunctive fairness (see Section 2.2) plays a role which, in case of polluted river problems, states that when agent i stops participation in an agreement with its downstream neighbor j (and i with all its upstream agents will make a new agreement on their own without the other agents, and the same for the component containing j), then the change in the contribution of i (and each of its complete dominating superiors) and j after breaking the agreement, should be equal. So, the refusal of an upstream neighbor of j to contribute to the cleaning cost in the river component with j affects j and the upstream neighbor by the same amount.

Before stating the axiom, we need to introduce some notation. Recall from the previous section that for river structure D and $(i,j) \in D$, $K^j_{ij}(D)$ is the component that is created after the deletion of the arc (i,j) and contains j. Next, we denote by $K^i_{ij}(D) = N \setminus K^j_{ij}(D) = \{i\} \cup \widehat{P}_D(i)$ the component that is created after the deletion of the arc (i, j) and contains agent i. Again, if there is no confusion about the river structure we denote $K^i_{ij}(D)$ just by K^i_{ij}. Note that i is the sink in the polluted river problem $(K^i_{ij}, D|_{K^i_{ij}}, c|_{K^i_{ij}})$ where $D|_{K^i_{ij}} = \{(h,k) \in D : \{h,k\} \subseteq K^i_{ij}\}$ is the river structure restricted to K^i_{ij}, and $c|_{K^i_{ij}}$ is the projection of the cost vector c on K^i_{ij}.

Participation fairness For every polluted river problem $(N, D, c) \in \mathcal{R}$ and $i, j \in N$ with $(i, j) \in D$ such that $|P_D(j)| \geq 2$, it holds that

$$g_j(K^j_{ij}, D|_{K^j_{ij}}, c|_{K^j_{ij}}) - g_j(N, D, c) = g_h(K^i_{ij}, D|_{K^i_{ij}}, c|_{K^i_{ij}}) - g_h(N, D, c), \tag{4.13}$$

for all $h \in \{i\} \cup \overline{P}_D(i)$.

Replacing externality fairness in Theorem 3.8 by participation fairness, characterizes the ULS method. Before proving this, we prove the following lemma.

Lemma 4.4. *If $F \subseteq S \subseteq N$, then $Q(F) \subseteq Q(S)$.*

Proof. Let $F \subseteq S \subseteq N$. If $F = \emptyset$, it is clear that $Q(F) = \emptyset \subset Q(S)$. Assume $F \neq \emptyset$, and assume there exists some $i \in Q(F) \setminus Q(S)$. If $i \notin Q(S)$ then $[i \notin S$ and $\emptyset \neq P_D(i) \nsubseteq Q(S)]$, or $[i \notin S$ and $P_D(i) = \emptyset]$. Since $i \notin S \Rightarrow i \notin F$, one has $\emptyset \neq P_D(i) \subseteq Q(F)$ from the assumption $i \in Q(F)$. Then there exists some $j \in P_D(i)$ such that $j \in Q(F) \setminus Q(S)$. Applying the same argument to j implies that there exists some $k \in P_D(j)$ such that $k \in Q(F) \setminus Q(S)$. One can repeat this argument infinitely many times, which then contradicts the fact that N is finite, and the fact that $k \in \widehat{P}_D(j)$ and $j \in P_D(i)$ implies that $i \notin P_D(k)$. Therefore, for every $i \in Q(F)$, it holds that $i \in Q(S)$, completing the proof. □

Proposition 4.5. *The ULS method satisfies efficiency, additivity, the inessential agent property, the necessary agent property, weak structural monotonicity and participation fairness.*

Proof. Efficiency, additivity, the inessential agent property, the necessary agent property and weak structural monotonicity, follow from (i) Algaba et al. (2003, Theorem 1), (ii) the fact that the set of disjunctive feasible coalitions in an acyclic digraph is an antimatroid,[16] and (iii) the definition of the ULS method as the disjunctive permission value of a game on a sink tree.[17]

To show participation fairness, note that the Shapley value also can be written using the Harsanyi dividends (Harsanyi, 1959) as

$$Sh_i(N, v) = \sum_{S \subseteq N : i \in S} \frac{\Delta_v(S)}{|S|},$$

where the Harsanyi dividend of coalition $S \subseteq N$ is given by $\Delta_v(\emptyset) = 0$ and recursively $\Delta_v(S) = v(S) - \sum_{T \subset S : T \neq S} \Delta_v(T)$ for $S \neq \emptyset$, which can be seen as the extra value that is generated by cooperation of the agents in S that was not yet generated by the proper subsets of S.

For any polluted river problem $(N, D, c) \in \mathcal{R}$ and $i, j \in N$ with $(i, j) \in D$ such that $|P_D(j)| \geq 2$, letting $w = r^d_{L^{sa}_{(N,D,c)}}$ and $w|_T(S) = w(S)$ for all $S \subseteq T$, and denoting $D' = D|_{K^j_{ij}}$, we can write

$$\begin{aligned}
& g^{ULS}_j(K^j_{ij}, D', c|_{K^j_{ij}}) - g^{ULS}_j(N, D, c) \\
&= \varphi^d(K^j_{ij}, L^{sa}_{(K^j_{ij}, D', c|_{K^j_{ij}})}, D') - \varphi^d(N, L^{sa}_{(N,D,c)}, D) = Sh_j(K^j_{ij}, r^d_{L^{sa}_{(K^j_{ij}, D', c|_{K^j_{ij}})}}) - Sh_j(N, r^d_{L^{sa}_{(N,D,c)}}) \\
&= \sum_{S \subseteq K^j_{ij} : j \in S} \frac{\Delta_{w|_{K^j_{ij}}}(S)}{|S|} - \sum_{S \subseteq N : j \in S} \frac{\Delta_w(S)}{|S|} \quad = \sum_{S \subseteq K^j_{ij} : j \in S} \frac{\Delta_w(S)}{|S|} - \sum_{S \subseteq N : j \in S} \frac{\Delta_w(S)}{|S|} \\
&= - \sum_{S \subseteq N : S \nsubseteq K^j_{ij}, j \in S} \frac{\Delta_w(S)}{|S|} \quad = - \sum_{S \subseteq N : S \cap K^i_{ij} \neq \emptyset, j \in S} \frac{\Delta_w(S)}{|S|},
\end{aligned} \tag{4.14}$$

[16] A set of feasible coalitions $\mathcal{A} \subseteq 2^N$ is an antimatroid (see Dilworth, 1940, Edelman and Jamison, 1985 and Korte et al., 1991) if it satisfies the following three properties: (i) $\emptyset \in \mathcal{A}$ (feasible empty set), (ii) $S, T \in \mathcal{A}$ implies that $S \cup T \in \mathcal{A}$ (union closedness), and (iii) $S \in \mathcal{A}$ with $S \neq \emptyset$, implies that there exists $i \in S$ such that $S \setminus \{i\} \in \mathcal{A}$ (accessibility).

[17] A direct proof using expression (4.12) instead of the two step definition of the ULS method as the disjunctive permission value of the associated restricted game, can be obtained from the authors on request.

where the third equality follows since $r^d_{L^{sa}_{(K^j_{ij}, D', c|_{K^j_{ij}})}}(S) = L^{sa}(\sigma^{D'}(S)) = L^{sa}(\sigma^D(S)) = r^d_{L^{sa}_{(N,D,c)}}(S)$ for all $S \subseteq K^j_{ij}$, and the fourth equality follows since $w(S) = w|_{K^j_{ij}}(S)$ for all $S \subseteq K^j_{ij}$.

Similarly, it can be shown that

$$g^{ULS}_i(K^i_{ij}, D|_{K^i_{ij}}, c|_{K^i_{ij}}) - g^{ULS}_i(N, D, c) = -\sum_{S \subseteq N: S \cap K^j_{ij} \neq \emptyset, i \in S} \frac{\Delta_w(S)}{|S|}. \tag{4.15}$$

Next, we define a coalition S to be *connected* if for all $i, j \in S$, it holds that one of the following three conditions is satisfied:

(i) $i \in \widehat{P}_D(j)$, or
(ii) $i \in \widehat{P}^{-1}_D(j)$, or
(iii) there is an $h \in S$ such that $h \in \widehat{P}^{-1}_D(i) \cap \widehat{P}^{-1}_D(j)$.

A coalition that is not connected is called *disconnected*.

To proceed with the proof, we need the following lemma whose proof can be found in Appendix C.

Lemma 4.6. *For every game $(N, v, D) \in \mathcal{GPR}$, the Harsanyi dividend $\Delta_{r^d_{v,D}}(S) = 0$ if $S \subseteq N$ is disconnected.*

Since (i) $S \notin \Phi^d_D$ implies $\Delta_w(S) = 0$ (see Algaba et al., 2003), (ii) $[S \in \Phi^d_D,\ S \cap K^i_{ij} \neq \emptyset, j \in S$, and S is connected] implies that $\{i, j\} \subseteq S$, (iii) $[S \in \Phi^d_D,\ S \cap K^j_{ij} \neq \emptyset, i \in S$, and S is connected] implies that $\{i, j\} \subseteq S$, and (iv) S is disconnected implies $\Delta_w(S) = 0$ (see Lemma 4.6), we have with (4.14) and (4.15) that

$$g^{ULS}_j(K^j_{ij}, D|_{K^j_{ij}}, c|_{K^j_{ij}}) - g^{ULS}_j(N, D, c) = g^{ULS}_i(K^i_{ij}, D|_{K^i_{ij}}, c|_{K^i_{ij}}) - g^{ULS}_i(N, D, c).$$

Since $[S \in \Phi^d_D$ and $i \in S]$ implies that $h \in S$ for all $h \in \overline{P}_D(i)$, we get that also

$$g^{ULS}_j(K^j_{ij}, D|_{K^j_{ij}}, c|_{K^j_{ij}}) - g^{ULS}_j(N, D, c) = g^{ULS}_h(K^i_{ij}, D|_{K^i_{ij}}, c|_{K^i_{ij}}) - g^{ULS}_h(N, D, c)$$

for all $h \in \overline{P}_D(i)$, showing that participation fairness is satisfied. □

Next we state the axiomatization of the ULS method.

Theorem 4.7. *The ULS method is the unique cost sharing method that satisfies efficiency, additivity, the inessential agent property, the necessary agent property, weak structural monotonicity and participation fairness.*

Proof. Proposition 4.5 shows that the ULS method satisfies all the axioms.

To show uniqueness, suppose that cost sharing method g satisfies the six axioms, and consider polluted river problem (N, D, c). We prove uniqueness of allocation method g for sink tree rivers by induction on the number of sources. We first show that for line-rivers the ULS method is uniquely determined by the first five axioms. A line-river has only one spring i_0 and satisfies $|P_D(i)| = 1$ for $i \in N \setminus \{i_0\}$. For any $i \in N$, again let $c^i \in \mathbb{R}^N_+$ be given by $c^i_i = c_i$ and $c^i_j = 0$ for all $j \in N \setminus \{i\}$. Similar to the proof of Theorem 3.5, the inessential agent property implies that $g_j(N, D, c^i) = 0$ for all $j \in N \setminus (\{i\} \cup \widehat{P}_D(i))$. By the necessary agent property,

$$g_i(N, D, c^i) \geq g_j(N, D, c^i) \text{ for all } j \in \widehat{P}_D(i). \tag{4.16}$$

By repeated application of weak structural monotonicity, it holds that

$$g_j(N, D, c^i) \geq g_i(N, D, c^i) \text{ for all } j \in \widehat{P}_D(i). \tag{4.17}$$

Equations (4.16) and (4.17) imply that $g_j(N, D, c^i) = g_i(N, D, c^i)$ for all $j \in \widehat{P}_D(i)$. Efficiency then determines that $g_i(N, D, c^i) = g_j(N, D, c^i) = \frac{c_i}{|\widehat{P}_D(i)|+1}$ for all $j \in \widehat{P}_D(i)$. Finally, additivity determines the payoffs according to the ULS method for any $c \in \mathbb{R}^N_+$ since $c = \sum_{i \in N} c^i$.

Proceeding by induction, assume that $g(N, D, c)$ is uniquely determined under the six axioms for all rivers with $|T(D)| \leq m$. For polluted river problems (N, D, c) with $|T(D)| = m + 1$, we will show that there are $|N|$ independent linear equations of $|N|$ unknown variables $g_i(N, D, c^\ell)$, $i \in N$, for each $\ell \in N$, which means that $g(N, D, c^\ell)$ is uniquely determined. Then $g(N, D, c)$ is obtained by additivity. Note that $|D| = |N| - 1$. We establish one equation associated with each arc in D. Since the river structure is a sink tree, every arc falls into one of the following cases:

(1) Suppose that $(i, j) \in D$ is such that $|P_D(j)| \geq 2$. Then from participation fairness we have

$$g_j(K^j_{ij}, D|_{K^j_{ij}}, c^\ell|_{K^j_{ij}}) - g_j(N, D, c^\ell) = g_i(K^i_{ij}, D|_{K^i_{ij}}, c^\ell|_{K^i_{ij}}) - g_i(N, D, c^\ell), \tag{4.18}$$

where $g_j(K^j_{ij}, D|_{K^j_{ij}}, c^\ell|_{K^j_{ij}})$ and $g_i(K^i_{ij}, D|_{K^i_{ij}}, c^\ell|_{K^i_{ij}})$ are already determined by the induction hypothesis because both river structures $(K^j_{ij}, D|_{K^j_{ij}})$ and $(K^i_{ij}, D|_{K^i_{ij}})$ have at most m springs.

(2) Suppose that $(i, j) \in D$ is such that $|P_D(j)| = 1$. This case further splits into two sub-cases:

(2-1) Suppose that the sink $L \notin \overline{P}_D^{-1}(i)$. Then there is an $h \in \overline{P}_D^{-1}(i)$ such that $|P_D(P_D^{-1}(h))| \geq 2$, i.e. the unique successor of h has at least two predecessors, including h. Let $P_D^{-1}(h) = \{k\}$. Then, deleting the arc (h, k), from participation fairness we have

$$g_k(K^k_{hk}, D|_{K^k_{hk}}, c^\ell|_{K^k_{hk}}) - g_k(N, D, c^\ell) = g_i(K^h_{hk}, h, D|_{K^h_{hk}}, c^\ell|_{K^h_{hk}}) - g_i(N, D, c^\ell), \tag{4.19}$$

where $g_k(K^k_{hk}, D|_{K^k_{hk}}, c^\ell|_{K^k_{hk}})$ and $g_i(K^h_{hk}, h, D|_{K^h_{hk}}, c^\ell|_{K^h_{hk}})$ are already determined by the induction hypothesis since river structure $(K^h_{hk}, D|_{K^h_{hk}})$ has at most m springs.

(2-2) Suppose that the sink $L \in \overline{P}_D^{-1}(i)$. In this case the equation depends on the location of agent ℓ. We consider again two sub-cases.

(2-2-1) If $\ell \in N \setminus \widehat{P}_D^{-1}(i)$, then by the inessential agent property it holds that

$$g_j(N, D, c^\ell) = 0; \tag{4.20}$$

(2-2-2) if $\ell \in \widehat{P}_D^{-1}(i)$, then by the necessary agent property and weak structural monotonicity, one has

$$g_i(N, D, c^\ell) = g_\ell(N, D, c^\ell). \tag{4.21}$$

The equations (4.18), (4.19) and (4.20) or (4.21)) yield $|D| = |N| - 1$ linear independent equations in the $|N|$ unknown variables $g_i(N, D, c^\ell)$, $i \in N$. Together with the last linear equation

$$\sum_{i \in N} g_i(N, D, c^\ell) = \sum_{i \in N} c_i^\ell = c_\ell,$$

which follows from efficiency, we can uniquely determine $g(N, D, c^\ell)$ for each $\ell \in N$. Additivity then uniquely determines $g(N, D, c)$ for all $(N, D, c) \in \mathcal{R}$. Since the ULS method satisfies these six axioms, g is the ULS method. □

Logical independence of the six axioms in Theorem 4.7 is again shown in Appendix B.

Compared with externality fairness, participation fairness equalizes the change in contribution between two agents if cooperation stops along the river segment between them. It is an expression of fairness where two agents are equally responsible when cooperation between them stops. In contrast, externality fairness expresses a fairness property between an agent and its remaining upstream neighbors when cooperation with one of its other upstream neighbors stops. This reflects that the agent and its remaining upstream neighbors are equally responsible for the additional cost caused by the withdrawal of one of its upstream neighbors. Participation fairness can be seen as an equity principle like upstream symmetry, but similar as externality fairness it does not equalize the payoffs of an agent and its downstream neighbor, but equalizes the change in their payoffs when they stop cooperation. So, it generates a different effect on the responsibility of an upstream agent to its downstream neighbors.

4.2. Comparison of the UES and ULS methods

We illustrate the ULS method and UES method by applying the ULS method to the example discussed in Section 3.4 of Dong et al. (2012), where the UES method is evaluated.

Example 4.8. This example models the Baiyangdian Lake Catchment in Northern China, see Dong et al. (2012) for details. The river structure and costs are depicted in Fig. 5, which is reproduced from Figure 3 of Dong et al. (2012). The solutions are summarized in Table 1.[18]

From Table 1, we can see that in this example, the ULS method allocates less costs to all the top agents compared to the UES method. In contrast, it allocates higher costs to the agent at the bottom. We can also see that for agents with a middle position (agents with both upstream and downstream neighbors), a difference of these two methods depends on their position in the river structure. For example, agent 3 shares higher costs in ULS than in UES. The intuition is that since agent 3 has two direct upstream neighbors, the costs generated at the bottom agent L is not clearly contributed by agent 1 or agent 2, but certainly passed through agent 3. Therefore, agent 3 is more responsible than its upstream agents regarding its downstream costs.

[18] We made some corrections in the calculation of the UES method as given in Dong et al. (2012).

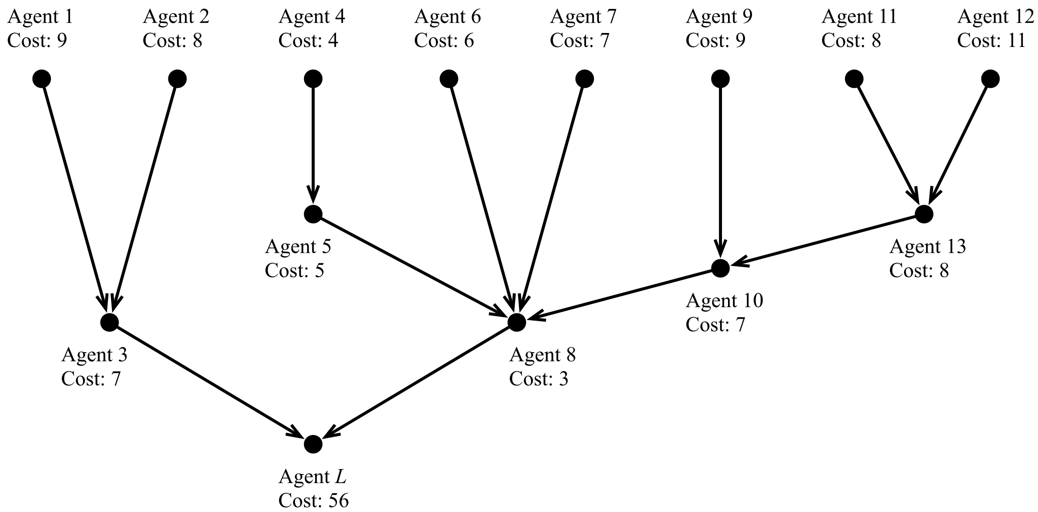

Fig. 5. The river structure and costs reproduced from Figure 3 of Dong et al. (2012).

Table 1
The ULS and UES solutions of the polluted river problem in Fig. 5.

Agent	ULS	UES	Agent	ULS	UES
1	12.2310	15.3333	8	11.0259	4.3000
2	11.2310	14.3333	9	10.6796	14.7000
3	11.7660	6.3333	10	4.8870	5.7000
4	6.8745	10.8000	11	9.6171	16.3667
5	2.8745	6.8000	12	12.6171	19.3667
6	7.3685	10.3000	13	6.2953	8.3667
7	8.3685	11.3000	L	32.1639	4.0000

In the example above, we see that the ULS method assigns higher cost shares than the UES method to the sink agent, and lower shares to the top agents. Whereas the ULS method can be seen as a compromise between the UES and LRS methods in the sense that, if there is only one agent with positive cost, then the contribution of this agent in its own cost, is always between its contribution according to the LRS and UES methods, it can be that top agents contribute more according to the ULS method than according to the UES method.

Proposition 4.9. *Consider polluted river problem (N, D, c) with $c_h = 0$ for all $h \in N \setminus \{i\}$. Then, $g_i^{LRS}(N, D, c) \geq g_i^{ULS}(N, D, c) \geq g_i^{UES}(N, D, c)$.*

Proof. Since the Limited Upstream-oriented game is monotone, and the ULS method is obtained as the Shapley value of this game, every agent pays a nonnegative contribution, and thus by efficiency $g_i^{LRS}(N, D, c) = c_i = L^{LU}_{(N,D,c)}(N) \geq g_i^{ULS}(N, D, c)$. To show the second inequality, notice that every coalition with a positive worth in $L^{U}_{(N,D,c)}$ contains agent i. and the same holds for $L^{LU}_{(N,D,c)}$. Moreover, every coalition has either worth zero or worth c_i. Since $\Phi^c_D \subseteq \Phi^d_D$, it holds that all marginal contributions of agent i in game $L^{LU}_{(N,D,c)}$ are greater or equal to its corresponding marginal contributions in $L^{U}_{(N,D,c)}$. Therefore, by definition of the Shapley value, $g_i^{ULS}(N, D, c) \geq g_i^{UES}(N, D, c)$. $\square$

By additivity, we can interpret Proposition 4.9 as every agent in the ULS method contributing in its own cost an amount between its contribution according to the LRS and UES methods. Since downstream agents always contribute zero to the costs of their upstream agents, we obtain the following corollary for the sink of a river structure.

Corollary 4.10. *Consider polluted river problem (N, D, c) with sink L. Then, $g_L^{LRS}(N, D, c) \geq g_L^{ULS}(N, D, c) \geq g_L^{UES}(N, D, c)$.*

Obviously, a top agent always contributes in the UES and ULS methods at least as much as in the LRS method. The following example shows that for a top agent the contribution according to the ULS method can be more than according to the UES method, depending on the river structure.

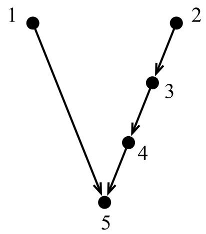

Fig. 6. A river with 5 agents.

Example 4.11. Consider the river problem (N, D, c) with $N = \{1, 2, 3, 4, 5\}$, river structure $D = \{(1, 5), (2, 3), (3, 4), (4, 5)\}$ (see Fig. 6) and cost vector c with $c_1 = c_2 = c_3 = c_4 = 0$ and $c_5 = 1$. This river has two top agents (agents 1 and 2) and a sink (agent 5). The allocation by the UES method and the ULS method are $g^{UES} = (\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})$ and $g^{ULS} = \frac{1}{20}(6, 1, 1, 1, 11)$.

We observe that agent 1, as a top agent, has to contribute more by the ULS method than by the UES method. Earlier, we already argued that uncertainty about where the pollution comes from when an agent has more upstream neighbors yields a higher contribution for this agent. In a similar way, uncertainty arising from more agents upstream of one of the downstream neighbors of an agent, in the figure agent 1 and its downstream neighbor 5, might result in a higher contribution of this agent. In the example, agent 4 might claim that the pollution it transfers to agent 5 is created by agents 2 and 3. (Similarly, agents 2 and 3 might claim that the pollution comes from other agents.) Agent 1 can only claim that the pollution at agent 5 came from the branch with agent 4, but whatever pollution entered the territory of agent 5 through the link with agent 1, must have been created by agent 1.

5. The DES method and the permission values

Besides the UES method, it is straightforward to see that the DES (Downstream Equal Sharing) method can be obtained as the conjunctive permission value of the game with permission structure $(N, L^{sa}_{(N,D,c)}, D^-)$ where $L^{sa}_{(N,D,c)}$ is the stand-alone game and the permission structure $D^- = \{(i, j) \in N \times N : (j, i) \in D\}$ is the down-upstream oriented digraph.

Moreover, since D^- is a rooted tree, and for rooted trees the conjunctive and disjunctive permission values coincide, the DES method is also obtained as the disjunctive permission value for the above mentioned game with permission structure.

Proposition 5.1. *Let $(N, D, c) \in \mathcal{R}$ be a polluted river problem. Then*

$$g^{DES}(N, D, c) = \varphi^c(N, L^{sa}_{(N,D,c)}, D^-) = \varphi^d(N, L^{sa}_{(N,D,c)}, D^-).$$

So, whereas applied to D the conjunctive and disjunctive permission value yield different cost sharing methods, applied to D^- both permission values yield the same cost sharing method, being the DES method.

Also in this case, axioms underlying the conjunctive (and disjunctive) permission value on rooted trees yield an axiomatization of the DES method.[19]

We say that a cost sharing method g is a *downstream oriented game method* if there is a solution f for games with a permission structure such that $g(N, D, c) = f(N, L^{sa}_{(N,D,c)}, D^-)$ for all $(N, D, c) \in \mathcal{R}$. Again, (i) efficiency for permission values on the class $\mathcal{GPR}^- = \{(N, v, D^-) \in \mathcal{GP} : v = L^{sa}_{(N,D,c)}$ for some $(N, D, c) \in \mathcal{R}\} = \{(N, v, D) \in \mathcal{GP} : v$ is inessential with $v(\{i\}) \geq 0$ for all $i \in N$ and D is a rooted tree $\} \subset \mathcal{GP}$ is equivalent to efficiency for polluted river cost sharing methods, and (ii) additivity for permission values on the class $\mathcal{GPR}^-$ is equivalent to additivity for polluted river cost sharing methods.

Since an agent i is an inessential player in a polluted river game with permission structure $(N, L^{sa}_{(N,D,c)}, D^-)$ if and only if its own cost as well as the cost of all its superiors in $\widehat{P}_D(i) = \widehat{P}^{-1}_{D^-}(i)$ is zero, the inessential player property for games with permission structure $(N, L^{sa}_{(N,D,c)}, D^-)$ is equivalent to requiring zero contributions for such agents.

D^--inessential agent property For every $(N, D, c) \in \mathcal{R}$ and $i \in N$ such that $c_j = 0$ for all $j \in \{i\} \cup \widehat{P}_D(i)$, it holds that $g_i(N, D, c) = 0$.

[19] The fairness axioms cannot be applied here since every agent (except the sink) has exactly one downstream neighbor, and thus $|\widehat{P}_{D^-}(i)| = |\widehat{P}^{-1}_D(i)| \leq 1$ for all $i \in N$.

Similar as for the inessential agent property, the D^--inessential agent property is stronger than independence of irrelevant costs since it also states requirements for the payoffs in polluted river problems where more than one agent has a positive cleaning cost, and requires that the cost of this agent is paid by this agent and its downstream agents.[20]

Proposition 5.2. *Every cost sharing method that satisfies the D^--inessential agent property also satisfies independence of irrelevant costs.*

Since the necessary player property for games with a permission structure does not relate to the permission structure, also for polluted river games with permission structure $(N, L^{sa}_{(N,D,c)}, D^-)$, the necessary player property is equivalent to the necessary agent property of Section 3.

Since stand-alone games are monotone, structural monotonicity on D^- is equivalent to requiring that downstream agents contribute at least as much as upstream agents.

D^--structural monotonicity For every $(N, D, c) \in \mathcal{R}$ and $i, j \in N$ with $j \in P^{-1}_{D^-}(i) = P_D(i)$, it holds that $g_i(N, D, c) \geq g_j(N, D, c)$.

Proposition 5.3. *Every cost sharing method that satisfies the necessary agent property and D^--structural monotonicity also satisfies downstream symmetry.*

Compared to Theorem 3.5, replacing the inessential agent property and structural monotonicity by the D^--inessential agent property and D^--structural monotonicity (or replacing independence of irrelevant costs, downstream symmetry and independence of downstream costs in Theorem 2.3 by the D^--inessential agent property, the necessary agent property and D^--structural monotonicity) characterizes the DES method. Similar as with Theorem 3.5, we do not need independence of downstream costs which is a rather strong axiom.

Theorem 5.4. *The DES method is the unique cost sharing method that satisfies efficiency, additivity, the D^--inessential agent property, the necessary agent property and D^--structural monotonicity.*

The alternative axioms in this section are related to Dong et al. (2012)'s Upstream Responsibility principle which they present as an alternative UTI interpretation compared to Downstream Responsibility.

6. Concluding remarks

In this paper we considered polluted river problems as games with a permission structure and showed how the UES and DES methods of Dong et al. (2012) can be obtained by applying the conjunctive permission value to an appropriate game with a permission structure. We also showed that axiomatizations of the conjunctive permission value yield new axiomatizations for the UES and DES methods. These axiomatizations have a good interpretation in terms of International Water Law. Also, we applied the disjunctive permission value to obtain a new cost sharing method, the ULS method, for polluted river problems.

This paper shows the strength of the Shapley value and its underlying axioms in applications such as polluted river cost allocation problems. In particular the interpretation of these axioms in terms of water allocation principles of International Water Law makes the Shapley value a very useful tool in developing cost sharing methods. The cost sharing methods discussed in this paper and the mentioned references are all based on the Shapley value, and differ with respect to the game that is considered. This makes the definition of the game an essential feature in developing cost sharing methods, but the elegance of the Shapley value guarantees some consistency in the developed cost sharing methods.

Although our goal was to stay within the framework of Dong et al. (2012) in the sense that we considered single sink rivers, we mention that the axiomatizations discussed in this paper hold for all strongly acyclic digraphs, being connected digraphs that might have multiple springs as well as multiple sinks, but from every agent there is a unique directed path to each of its downstream agents, see Fig. 7. The axioms can be defined similar as they are before, and the proofs follow similar arguments. Proof of uniqueness follows similarly as in the proofs of Theorems 3.5, 3.8 and 4.7 by considering the cost vectors c^i, $i \in N$, where only one agent has a positive cost, e.g. the black dotted agent in Fig. 7. Since all agents $j \neq i$ that are not upstream of i pay zero in c^i by the inessential agent property, considering the river structure on i and all its upstream agents is, in fact, a sink tree and we can apply the axioms similarly as in the proofs of Theorems 3.5, 3.8 and 4.7.

The Shapley value is one of the best known solutions for cooperative games. This is not a coincidence. In previous literature it has been shown that the Shapley value has appealing properties, not only for classical cooperative games, but also for games with restricted cooperation such as communication graph games (see Myerson, 1977), games with a

[20] The proofs of Propositions 5.2 and 5.3 and Theorem 5.4 go similar as the proofs of Propositions 3.3 and 3.4 and Theorem 3.5. They are therefore omitted. The proofs can be obtained from the authors on request.

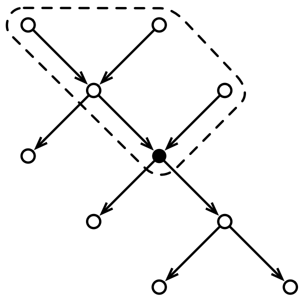

Fig. 7. A strongly acyclic river with multiple springs and multiple sinks.

permission structure (see Gilles et al., 1992) and many of their generalizations. Its elegant properties have been extended to several generalized models in the literature, but also do very well in applied profit and cost sharing methods, as illustrated in this paper for polluted river problems. This makes the Shapley value one of the most useful solutions for cooperative games.

Appendix A

The following cost sharing methods show that neither the necessary agent property nor structural monotonicity on its own implies upstream symmetry, and upstream symmetry implies neither the necessary agent property nor structural monotonicity.

1. The DES method satisfies the necessary agent property but does not satisfy upstream symmetry.
2. Consider the method

$$g_i(N,D,c)=\begin{cases}\frac{\sum_{h\in N}c_h}{|T(D)|} & \text{if } i\in T(D)\\ 0 & \text{otherwise,}\end{cases}$$

which equally allocates the full cleaning cost in the river over the most upstream agents. This method satisfies structural monotonicity but does not satisfy upstream symmetry.
3. Consider the modified DES method given by

$$g_i^{\overline{DES}}(N,D,c)=\sum_{j\in\widehat{P}_D(i)}\frac{c_j}{\left|\widehat{P}_D^{-1}(j)\right|}+\frac{c_L}{|N|}$$

where the cost of every river segment is equally shared among all agents downstream of the segment (so compared to the DES method the upstream agent on a river segment does not contribute to the cleaning costs), and the cost of the sink is equally shared among all agents. This method satisfies upstream symmetry, but it does not satisfy the necessary agent property nor structural monotonicity.

Appendix B

Logical independence of the five axioms in Theorem 3.5 can be seen from the following alternative cost sharing methods:

1. The Local Responsibility Sharing method satisfies all axioms except structural monotonicity.
2. Consider the modified UES method given by

$$g_i^{\overline{UES}}(N,D,c)=\begin{cases}\sum_{j\in\widehat{P}_D^{-1}(i)}\frac{c_j}{\left|\widehat{P}_D(j)\right|} & \text{if } i\notin T(D)\\ \sum_{j\in\widehat{P}_D^{-1}(i)}\frac{c_j}{\left|\widehat{P}_D(j)\right|}+c_i & \text{if } i\in T(D)\end{cases}$$

where the cost of every river segment is equally shared among all agents upstream of the upstream agent on the segment (so compared to the UES method the upstream agent on a river segment does not contribute to the cleaning costs).[21] In the case that the upstream agent of a river segment is a top agent, the cost of this agent is allocated to itself. This method satisfies all axioms except the necessary agent property.

[21] This is a modification of the UES method, similar as the DES method is modified to $g^{\overline{DES}}$ in Appendix A.

3. Consider the method

$$g_i^{eq}(N,D,c) = \frac{\sum_{h\in N} c_h}{|N|} \text{ for all } i \in N$$

where the full cleaning cost is equally shared among all agents. This method satisfies all axioms except the inessential agent property.

4. Consider the method which allocates as the UES method in the case that there is a necessary agent (that is, when a single agent has non-zero cleaning cost), and allocates as the $\overline{\text{UES}}$ method $g^{\overline{UES}}$ otherwise (that is, when there is more than one agent having non-zero cleaning cost). This method satisfies all axioms except additivity.[22]
5. Consider the method given by

$$g_i^{zero}(N,D,c) = 0 \text{ for all } i \in N$$

This method satisfies all axioms except efficiency.

Logical independence of the six axioms in Theorem 3.8 can be seen from the following alternative cost sharing methods:

1. The Upstream Limited Sharing method (see Section 4) satisfies all axioms except externality fairness.
2. The Local Responsibility Sharing method satisfies all axioms except weak structural monotonicity.
3. Consider the method given by

$$g_i(N,D,c) = \begin{cases} \sum_{j\in \widehat{P}_D^{-1}(i)} \frac{c_j(1+\frac{1}{|N|})}{|\widehat{P}_D(j)|+1} + c_i\left(\frac{1+\frac{1}{|N|}}{|\widehat{P}_D(i)|+1} - \frac{1}{|N|}\right) & \text{if } i \notin T(D) \\ \sum_{j\in \widehat{P}_D^{-1}(i)} \frac{c_j(1+\frac{1}{|N|})}{|\widehat{P}_D(j)|+1} + c_i & \text{if } i \in T(D) \end{cases}$$

where the cost of every river segment is unequally shared among all agents that are located upstream of that segment, such that each agent upstream of the local agent always shares a fixed portion more than the local agent. In the case that the local (upstream) agent of a river segment is a top agent, the cost of this agent is allocated to itself. This method satisfies all axioms except the necessary agent property.[23]
4. The method g^{eq} (see method 3 above) that equally assigns the full cleaning cost among all agents satisfies all axioms except the inessential agent property.
5. Consider the method that allocates as UES when there is a necessary agent and allocates as method stated in 3 otherwise. This method satisfies all axioms except additivity.
6. The method g^{zero} (see method 5 above) that assigns zero costs to all agents satisfies all axioms except efficiency.

Logical independence of the six axioms in Theorem 4.7 can be seen from the following alternative cost sharing methods:

1. The UES method satisfies all axioms except participation fairness.
2. The LRS method satisfies all axioms except weak structural monotonicity.
3. Consider the method that allocates all the costs to its top agent with line rivers. And with non-line rivers, we reassign the initial costs in the following way. For every agent i with two or more predecessors, we reassign the costs of the agents that are completely dominated by i to agent i. After this reassignment, we apply efficiency, additivity, the inessential agent property, participation fairness together with the line river cases to the new problem. This method leads to a unique sharing outcome that satisfies all axioms except the necessary agent property.
4. Consider the method that equally assigns the full cleaning costs among all agents with line rivers. And with non-line rivers, given the allocation results in the linear case, it restricts one more condition in addition to efficiency, additivity, the necessary agent property and participation fairness. This condition requires that for every agent with only one direct upstream neighbor, it always pays the same as its upstream neighbor. This method provides a unique sharing outcome that satisfies all axioms except the inessential agent property.
5. Consider the method that allocates as ULS when there is a necessary agent and allocates as the method stated in 3 otherwise. This method satisfies all axioms except additivity.
6. The method g^{zero} that assigns zero costs to all agents satisfies all axioms except efficiency.

[22] Note that the necessary agent property only states a requirement if there is a single agent who has non-zero cleaning cost at its downstream river segment. So by allocating the cost in a different way from UES when a necessary agent is absent, additivity is violated.

[23] Without getting into details, it can be proved that (i) the axioms determine a unique method, and (ii) this method satisfies the other axioms. Uniqueness follows by induction, starting with line river games where equal allocation is applied, and by induction on the number of springs, for each non-spring agent i with $|P_D(i)| = 1$ weak structural monotonicity yields one equation, while in case $|P_D(i)| > 1$, participation fairness yields $|P_D(i)| - 1$ equations. Together with efficiency these are $|N|$ linear independent equations yielding a unique method. Also by induction on the number of springs, it can be shown that all cost shares are nonnegative and weak structural monotonicity is satisfied. The other axioms are obvious.

Appendix C

The following lemma is used in the proof of Proposition 4.5 in Section 4.

Lemma 4.6. *For any game $(N, v, D) \in \mathcal{GPR}$, the Harsanyi dividend $\Delta_{r^d_{v,D}}(S) = 0$ if $S \subseteq N$ is disconnected.*

Proof. It follows from Algaba et al. (2003) that $S \notin \Phi^d_D$ implies $\Delta_{r^d_{v,D}}(S) = 0$, therefore we only need to consider coalitions $S \in \Phi^d_D$. Here we say $R \subseteq S$ is a maximal connected part of S if there exists no other connected $R' \subseteq S$ such that $R \subset R'$ and $R \neq R'$. Let $H(S) = \{R \subseteq S : R \text{ is a maximal connected part of } S\}$. Obviously $H(S)$ is a partition of S. If $S \in \Phi^d_D$, then $r^d_{v,D}(S) = \sum_{i\in S} v(\{i\})$. For any $R \in H(S)$, it holds that $R \in \Phi^d_D$ and thus $r^d_{v,D}(R) = \sum_{i\in R} v(\{i\})$. It is easy to see that $\Delta_{r^d_{v,D}}(S) = 0$ for disconnected $S \in \Phi^d_D$ with $|S| = 2$, since $H(S)$ contains two singletons and $\Delta_{r^d_{v,D}}(\{i\}) = v(\{i\})$ for any $i \in N$.[24] Assume for some $m > 2$ that $\Delta_{r^d_{v,D}}(S) = 0$ holds true for disconnected $S \in \Phi^d_D$ with $|S| \leq m$, then the Harsanyi dividend $\Delta_{r^d_{v,D}}(S)$ of disconnected $S \in \Phi^d_D$ with $|S| = m + 1$ can be written as

$$\begin{aligned}\Delta_{r^d_{v,D}}(S) &= r^d_{v,D}(S) - \sum_{T\subset S: T\neq S} \Delta_{r^d_{v,D}}(T)\\ &= \sum_{i\in S} v(\{i\}) - \sum_{T\subset S: T\neq S} \Delta_{r^d_{v,D}}(T)\\ &= \sum_{R\in H(S)} \Big[\sum_{i\in R} v(\{i\}) - \sum_{T\subset R: T\neq R} \Delta_{r^d_{v,D}}(T) - \Delta_{r^d_{v,D}}(R)\Big]\\ &= \sum_{R\in H(S)} \Big[\Delta_{r^d_{v,D}}(R) - \Delta_{r^d_{v,D}}(R)\Big] \quad = 0,\end{aligned}$$

where the third equality follows from $|R| < |S|$ for $R \in H(S)$ and the induction hypothesis. □

References

Alcalde-Unzu, J., Gómez-Rúa, M., Molis, E., 2015. Sharing the costs of cleaning a river: the Upstream responsibility rule. Games Econ. Behav. 90, 134–150.
Algaba, E., Bilbao, J.M., van den Brink, R., Jiménez-Losada, A., 2003. Axiomatizations of the Shapley value for cooperative games on antimatroids. Math. Methods Oper. Res. 57 (1), 49–65.
Ambec, S., Ehlers, L., 2008. Sharing a river among satiable agents. Games Econ. Behav. 64 (1), 35–50.
Ambec, S., Sprumont, Y., 2002. Sharing a river. J. Econ. Theory 107 (2), 453–462.
Ansink, E., Weikard, H.P., 2012. Sequential sharing rules for river sharing problems. Soc. Choice Welfare 38 (2), 187–210.
van den Brink, R., 1997. An axiomatization of the disjunctive permission value for games with a permission structure. Int. J. Game Theory 26, 27–43.
van den Brink, R., 1999. An axiomatization of the conjunctive permission value for games with a hierarchical permission structure. In: de Swart, H. (Ed.), Logic, Game theory and Social Choice. In: Proceedings of the International Conference LGS '99. Tilburg University Press, pp. 125–139.
van den Brink, R., 2001. An axiomatization of the Shapley value using a fairness property. Int. J. Game Theory 30, 309–319.
van den Brink, R., Gilles, R.P., 1996. Axiomatizations of the conjunctive permission value for games with permission structures. Games Econ. Behav. 12, 113–126.
van den Brink, R., van der Laan, G., Moes, N., 2012. Fair agreements for sharing international rivers with multiple springs and externalities. J. Environ. Econ. Manage. 63 (3), 388–403.
van den Brink, R., van der Laan, G., Vasil'ev, V.A., 2007. Component efficient solutions in line-graph games with applications. Econ. Theory 33 (2), 349–364.
van den Brink, R., van der Laan, G., Vasil'ev, V.A., 2014a. Constrained core solutions for totally positive games with ordered players. Int. J. Game Theory 43, 351–368.
van den Brink, R., Estévez-Fernández, A., van der Laan, G., Moes, N., 2014b. Independence of downstream and upstream benefits in river water allocation problems. Soc. Choice Welfare 43 (1), 173–194.
Dilworth, R.P., 1940. Lattices with unique irreducible decompositions. Ann. Math. 41 (4), 771–777.
Dong, B., Ni, D., Wang, Y., 2012. Sharing a Polluted River Network. Environ. Resource Econ. 53 (3), 367–387.
Edelman, P.H., Jamison, R.E., 1985. The theory of convex geometries. Geom. Dedic. 19 (3), 247–270.
Gilles, R.P., Owen, G., 1994. Games with permission structures: the disjunctive approach. Department of Economics Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Mimeo.
Gilles, R.P., Owen, G., van der Brink, R., 1992. Games with permission structures: the conjunctive approach. Int. J. Game Theory 20, 277–293.
Harsanyi, J.C., 1959. A bargaining model for cooperative *n*-person games. In: Tucker, A.W., Luce, R.D. (Eds.), Contributions to the Theory of Games IV. Princeton University Press, Princeton, pp. 325–355.
Hu, X., Shapley, L.S., 2003. On authority distributions in organizations: controls. Games Econ. Behav. 45, 153–170.
International Law Association, 1966. The Helsinki Rules. Report of the Fifty-Second Conference, pp. 484–533.
Kalai, E., Samet, D., 1987. On weighted Shapley values. Int. J. Game Theory 16 (3), 205–222.
Khmelnitskaya, A.B., 2010. Values for rooted-tree and sink-tree digraph games and sharing a river. Theory Dec. 69 (4), 657–669.
Kilgour, D.M., Dinar, A., 1995. Are Stable Agreements for Sharing International River Waters Now Possible? Policy Research Working Paper 1474. World Bank.

[24] This holds only for inessential games but not for general games.

Korte, B., Lovász, L., Schrader, R., 1991. Greedoids. Algorithms Comb., vol. 4. Springer-Verlag, Berlin.
Lipper, J., 1967. Equitable utilization. In: Garretson, A.H., Hayton, R.D., Olmstead. Dobbs Ferry, C.J. (Eds.), The Law of International Drainage Basins. Oceana Publications Inc., New York, pp. 15–88.
McCaffrey, S.C., 2001. The Law of International Watercourses. Oxford University Press, Oxford.
Myerson, R.B., 1977. Graphs and cooperation in games. Math. Oper. Res. 2 (3), 225–229.
Ni, D., Wang, Y., 2007. Sharing a polluted river. Games Econ. Behav. 60, 176–186.
Parrachino, I., Dinar, A., Patrone, F., 2006. Cooperative Game Theory and its Application to Natural, Environmental, and Water Resource Issues: 3. Application to Water Resources. World Bank Policy Research Working Paper No. 4074.
Shapley, L.S., 1953. A value for n-person games. In: Kuhn, H.W., Tucker, H.W. (Eds.), Contributions to the Theory of Games, Volume II. In: Ann. Math. Stud., vol. 28. Princeton University Press, Princeton, New Jersey, pp. 307–317.
Wang, Y., 2011. Trading water along a river. Math. Soc. Sci. 61 (2), 124–130.